ECE 409 - SIGNAL SPACE ANALYSIS - INVESTIGATION 16
VECTOR REPRESENTATION OF SIGNALS

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

From the last two Investigations we know that QPSK signals can be written as follows

$$s_{QPSK}(t) = x_1(t)\phi_1(t) + x_2(t)\phi_2(t) = x_1(t)\sqrt{\frac{2}{T}}\cos(2\pi f_c t) + x_2(t)\sqrt{\frac{2}{T}}\sin(2\pi f_c t)$$

where $x_1(t)$ and $x_2(t)$ are polar NRZ_L message signals of amplitude $\sqrt{E/2}$ and

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t) \quad \text{and} \quad \phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t)$$

are the basis functions. We say that QPSK signals form a 2-dimensional signal space because every signal can be written as a sum of $\phi_1(t)$ and $\phi_2(t)$

Now as we more extensively vary the parameters of our sinusoids and pulses we need a more general method for designing our detectors. The objective of this and the next two Investigations on signal space analysis is to show how expressing signals in terms of basis functions

(1) Not only gives us a convenient way to generate more general signals

(2) But also facilitates the design of optimal detectors - detectors that choose the signal s_k that maximizes the following conditional probability

$$P(s_k | r)$$

the signal that was most likely to have been transmitted when we receive the signal r

The objective of this Investigation in particular is to find additional examples of signals that form 2-dimensional signal spaces and then generalize to n-dimensional signal spaces

1. We begin with some familiar binary signals. Express each of the following binary signals in terms of orthonormal basis functions and then plot their constellations
   a. BASK
   b. BFSK
   c. BPSK

2. This problem introduces a new 2-dimensional signal space consisting of 8-phase shift keying (8PSK) signals of the following form

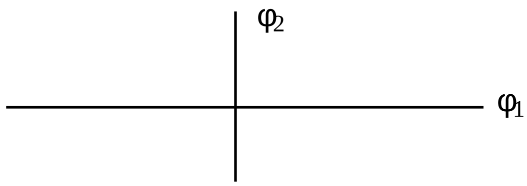
$$s_i(t) = \sqrt{\frac{2E}{T}}\cos\left[2\pi f_c t + (i-1)\frac{\pi}{4}\right] \qquad 0 \le t \le T \qquad i = 1,2,\ldots,8$$

   with f_c an integer multiple of 1/T and basis functions

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t) \qquad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t) \qquad 0 \le t \le T$$

a. Show that $s_{i1} = \sqrt{E}\cos\left[(i-1)\frac{\pi}{4}\right]$ and $s_{i2} = -\sqrt{E}\sin\left[(i-1)\frac{\pi}{4}\right]$
b. Calculate $s_i = (s_{i1}, s_{i2})$ for $i = 1, 2, \ldots, 8$
c. Verify that all the vectors s_i have the same amplitude
d. Plot the constellation of points $s_1, s_2, \ldots, s_8$ on the following 2-dimensional vector space



e. Describe your constellation of points in part (d)

3. The objective of this problem is to combine ASK with PSK to form the very popular 2-dimensional **Quadrature Amplitude Modulation (QAM)** signals as follows

$$s_i(t) = A_{ic}\sqrt{E}\sqrt{\frac{2}{T}}\cos(2\pi f_c t) - A_{is}\sqrt{E}\sqrt{\frac{2}{T}}\sin(2\pi f_c t) \qquad 0 \le t \le T \qquad i = 1, 2, \ldots, 8$$

with

$$A_{ic} = \begin{cases} -3 & i = 1,2 \\ -1 & i = 3,4 \\ 1 & i = 5,6 \\ 3 & i = 7,8 \end{cases} \qquad A_{is} = \begin{cases} -1 & i = 1,3,5,7 \\ 1 & i = 2,4,6,8 \end{cases}$$

a. Find and sketch the first five signals $s_i(t)$
b. What are the amplitudes of your signals in part (a)
c. Plot the constellation of this QAM signal

4. We begin our generalization to n-dimensional signal spaces by generalizing our definition of orthonormal. We say that a set of N signals $\{\phi_i(t), \quad i = 1, 2, \ldots, N\}$ is **orthonormal** if

$$\int_0^T \phi_j(t)\phi_k(t)\,dt = \begin{cases} 1 & j = k \\ 0 & j \ne k \end{cases}$$

Describe in words what it means for a set of signals to be orthonormal

5. The objective of this problem is to show that the following set of time shifted rectangles

$$\phi_i(t) = \sqrt{\frac{N}{T}}\ \mathrm{rect}\left(\frac{t - \frac{(2i-1)T}{2N}}{\frac{T}{N}}\right) \qquad i = 1,\ldots,N$$

is orthonormal where $\mathrm{rect}\left(\frac{t-a}{b}\right)$ is a pulse of amplitude one and width b that is centered at a

a. First sketch $\phi_i(t)$ on separate graphs for $i = 1,2,3$
b. Describe your graphs in part (a) - do the pulses overlap
c. Then show that the $\phi_i(t)$ are orthonormal

6. The objective of this Investigation is to show that the following sinusoids are orthonormal

$$\phi_i(t) = \begin{cases} \sqrt{\frac{2}{T}} \cos\left[2\pi(f_c + (i-1)\Delta f)t\right] & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

a. Sketch the first three sinusoids assuming that $\Delta f = \frac{1}{2T}$
b. Describe how the sinusoids are different from each other
c. Show that the sinusoids are orthonormal

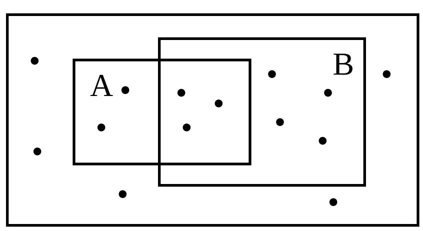
7. Draw a block diagram to illustrate how a signal $s_i(t)$ as follows

$$s_i(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t) + \cdots + s_{iN}\phi_N(t)$$

can be generated from the s_{ik}'s and basis functions $\phi_1(t), \phi_2(t), \ldots, \phi_N(t)$

8. Make use of the fact that the basis functions are orthonormal to obtain a block diagram of a circuit for recovering the coefficients $s_{i1}, s_{i2}, \ldots, s_{iN}$

9. The objective of this problem is to review conditional probability. Suppose we do a random experiment a whole bunch of times and get results as follows



Find the following probabilities

a. $P(A)$
b. $P(A \mid B)$
c. Describe in words what conditional probability is

10. What is meant by *a priori* and *a posteriori* probabilities. Illustrate with an example