

# ECE 306 - THE VERY BASICS - INVESTIGATION 5 INTRODUCTION TO RECURSIVE AND NONRECURSIVE DIFFERENCE EQUATIONS

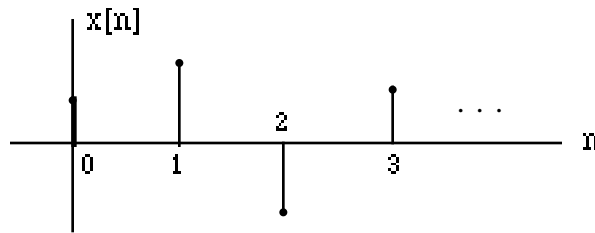
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From Investigation (1) we know that digital signal processing typically involves calculating the responses of linear difference equations with constant coefficients like the following

$$y[n] = 0.5y[n - 1] + x[n] - 1.7x[n - 1]$$

to discrete inputs like this one



obtained by sampling a continuous input  $x(t)$ . The main objective of this Investigation is to learn more about these difference equations - in particular how they respond to discrete step functions  $u[n]$  and discrete impulses  $\delta[n]$ .

1. Difference equations like the following

$$y[n] = 0.5y[n - 1] + x[n] - 1.7x[n - 1]$$

where  $y[n]$  depends not only on the input  $x[n]$  but also previous values of  $y[n]$  are referred to as **recursive**. On the other hand, difference equations like this one

$$y[n] = x[n] - 1.7x[n - 1]$$

where  $y[n]$  depends only on present and previous values of  $x[n]$  are referred to as **nonrecursive**. **Memorize** these definitions. Then

- a. Make up your own example of a recursive difference equation
- b. Make up your own example of a nonrecursive difference equation

2. The **order** of a difference equation is by definition the *larger* of the following:

- (1) The number of registers needed to store previous values of  $x[n]$
- (2) The number of registers need to store previous values of  $y[n]$

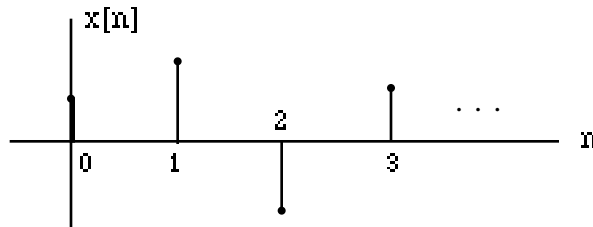
**Memorize** this definition. Then find the order of

- a.  $y[n] = x[n] - 1.7x[n - 1]$
- b.  $y[n] = 0.5y[n - 1] + x[n] - 1.7x[n - 1]$
- c.  $y[n] = 2x[n - 2]$

3. The objective of this and the rest of the problems of this investigation is to calculate the responses of some recursive and nonrecursive discrete system to some common inputs like discrete steps and impulses. We begin by defining the **discrete step function  $u[n]$**  as follows

$$u[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

- a. Sketch a graph of  $u[n]$  as a function of  $n$ . Remember that discrete plots look as follows



- b. Describe in words what  $u[n]$  looks like.  
 c. Sketch  $2u[n]$   
 d. Sketch  $2u[n - 2]$   
 e. Sketch  $u[n] - u[n - 1]$   
 f. What would you expect is meant by the step response of a discrete system
4. The objective of this problem is to calculate and sketch the **step responses** of some nonrecursive discrete systems - the responses when
- (1) The input  $x[n]$  equals the unit step  $u[n]$
  - (2) All initial conditions  $y[-1], y[-2], \dots$  are zero

**Memorize** this definition. Then for each of the following first order discrete systems find and sketch the step responses from  $n = 0$  to  $n = 5$ . Note that we refer to step responses as  $s[n]$

- a.  $y[n] = x[n]$
  - b.  $y[n] = x[n] - x[n - 1]$
  - c.  $y[n] = -x[n] + 2x[n - 1]$
  - d. Explain why the step responses of nonrecursive difference equations like those in parts (a) to (c) reach steady state after only a finite number of sample times.
5. The objective of this problem is to see how the coefficient of  $y[n - 1]$  in a first order recursive difference equation affects its step response. Calculate and plot the step responses of each of the following from  $n = 0$  to  $n = 5$
- a.  $y[n] = 0.5y[n - 1] + x[n]$
  - b.  $y[n] = -0.5y[n - 1] + x[n]$
  - c.  $y[n] = y[n - 1] + x[n]$
  - d.  $y[n] = -y[n - 1] + x[n]$
  - e.  $y[n] = 2y[n - 1] + x[n]$
  - f.  $y[n] = -2y[n - 1] + x[n]$
6. From the previous two problems we see that while the step responses of nonrecursive systems are pretty sedate, the step responses of first order recursive systems can not only oscillate but also go off to infinity. What can be shown is that the step response of a general first order difference equation as follows

$$y[n] = ay[n - 1] + bx[n] + cx[n - 1]$$

will reach a steady state value if  $|a| < 1$  but will go off to infinity if  $|a| > 1$ . Find the constant value that the step response of the following difference equation

$$y[n] = 0.8y[n-1] + 2x[n] + x[n-1]$$

is approaching as  $n$  goes to infinity. Hint - for large  $n$ ,  $y[n] = y[n-1]$

7. The objective of this problem is to define the **discrete impulse** or **Kronecker Delta function**  $\delta[n]$  as follows

$$\delta[n] = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{otherwise} \end{cases}$$

- Plot  $\delta[n]$ . Describe in words what  $\delta[n]$  looks like
  - Plot  $2\delta[n]$
  - Plot  $2\delta[n-1]$
  - Plot  $\delta[n] + \delta[n-1] + \delta[n-2]$
  - What would you expect is meant by the impulse response of a discrete system.
8. The objective of this problem is to calculate and sketch the **impulse responses** of some recursive and nonrecursive discrete systems - the responses when

- The input  $x[n]$  equals the impulse  $\delta[n]$
- All initial conditions  $y[-1], y[-2], \dots$  are zero

**Memorize** this definition. Then find and sketch the impulse responses of the following discrete systems. Note that we refer to the values of the impulse response as  $h[n]$

- $y[n] = x[n] - 1.7x[n-1]$
  - $y[n] = 0.5y[n-1] + x[n] - 1.7x[n-1]$
  - Describe the difference between the impulse responses of the recursive and nonrecursive difference equations in parts (a) and (b) above
9. Given the following nonrecursive digital filter

$$y[n] = x[n] - 1.7x[n-1] + 0.5x[n-2]$$

- Find the values of the impulse response  $h[n]$
  - Describe how the values of  $h[n]$  are related to the coefficients of the nonrecursive digital filter. **Memorize** this result.
10. If the impulse response  $h[n]$  of a digital filter becomes zero after only a finite number of samples - as it does for nonrecursive filters - then we say the filter is a **Finite Impulse Response** or **FIR** filter. If, on the other hand, the impulse response goes on forever as it does for the recursive filters we've been working with then we say the filter is an **Infinite Impulse Response** or **IIR** filter. **Memorize** these definitions. Then
- Write the difference equation of an FIR filter. Show that it is in fact FIR
  - Write the difference equation of an IIR filter. Show that it is in fact IIR
11. Math Review: Express each of the following as a cosine
- $x(t) = 2e^{-j2t} + 2e^{j2t}$
  - $x(t) = 2e^{-j(2t+1.2)} + 2e^{j(2t+1.2)}$