ECE 306 - POLES AND ZEROS - INVESTIGATION 25
MORE POLES AND ZEROS - PART II

WINTER 2007 **A.P. FELZER**

In the last Investigation we reviewed the fact that the zeros of transfer functions like the following

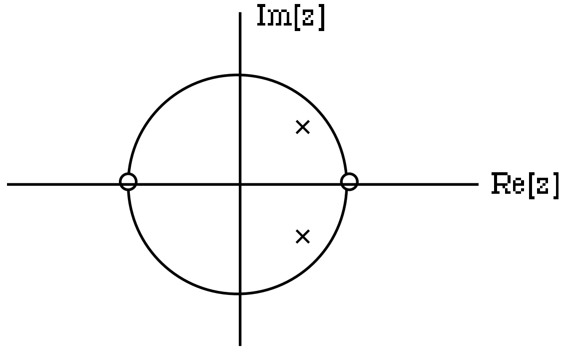
$$H(z) = \frac{z(z-1)}{(z+0.5)(z+0.4)}$$

are the roots of the numerator and the poles are the roots of the denominator. And so in general we can write

$$H(z) = K\frac{(z-z_1)(z-z_2)\cdots(z-z_m)}{(z-p_1)(z-p_2)\cdots(z-p_n)}$$

for transfer functions H(z) with m zeros and n poles. We also demonstrated the fact that the terms in the natural response are directly related to the poles. The objective of this Investigation is to review how the locations of the poles in the complex plane are related to the natural response as first introduced in Investigation 9.

1. We begin with a review of pole-zero plots for transfer functions H(z) like the following



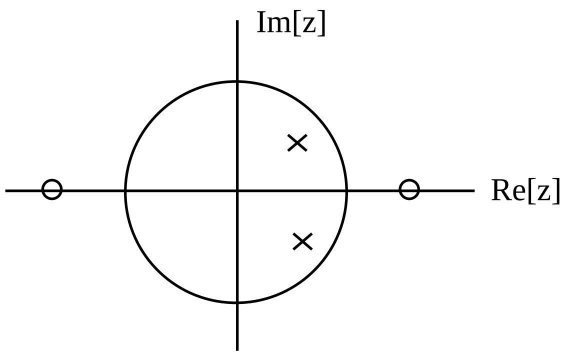
with poles × and zeros ○. Draw the pole-zero diagram for discrete systems with

a. $z_1 = 0.5, \quad z_2 = 0, \quad p_1 = -0.5 + j0.3, \quad p_2 = -0.5 - j0.3$

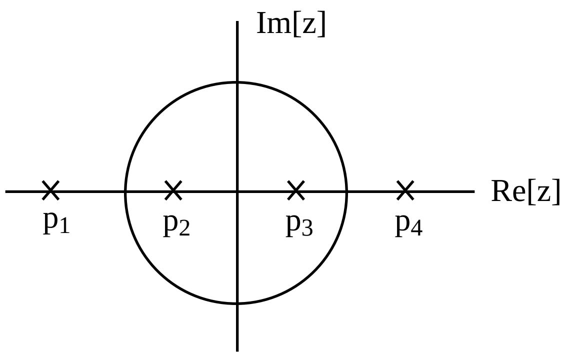
b. $H(z) = 2\,\dfrac{(z+1)(z-1)}{(z-0.5+j0.4)(z-0.5-j0.4)}$

c. $y[n] = 0.5y[n-1] - 0.3y[n-2] + x[n] - 0.5x[n-1]$

2. Why do we always draw pole-zero diagrams of discrete systems with unit circles of radius one as follows



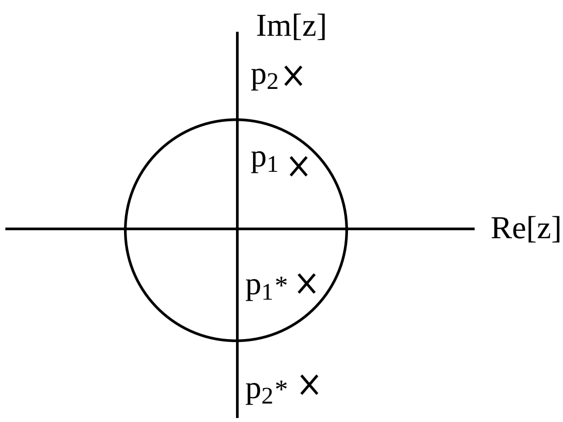
3. Given a discrete system with the following pole-zero diagram



Sketch the natural responses corresponding to the poles

a. p_1
b. p_2
c. p_3
d. p_4

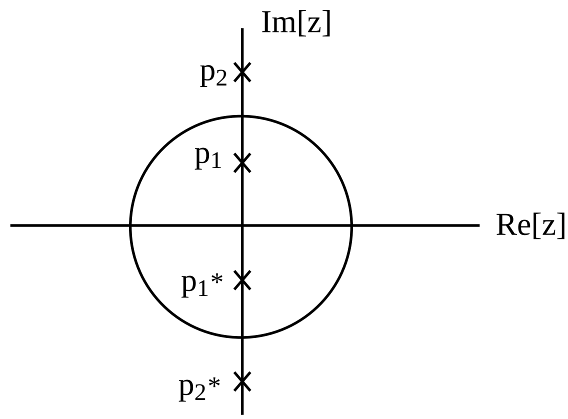
4. Given a discrete system with the following pole-zero diagram



Sketch the natural responses corresponding to the poles

a. p_1, p_1^*
b. p_2, p_2^*

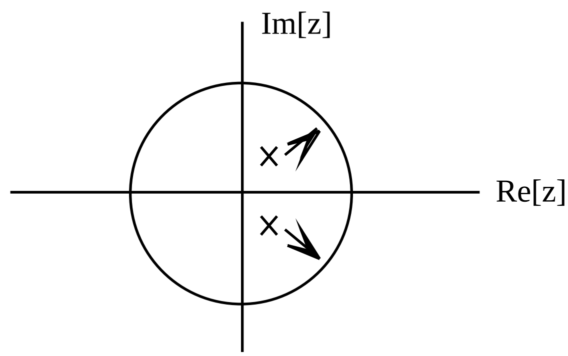
5. Given a discrete system with the following pole-zero diagram



Sketch the natural responses corresponding to the poles

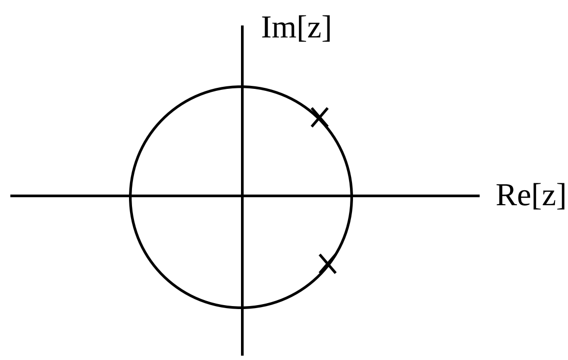
a. p_1, p_1*
b. p_2, p_2*

6. What happens to the time it takes the natural response to decay as the poles approach the unit circle as follows



Illustrate with an example. **Memorize** this result

7. What happens to the natural response when the poles are right on the unit circle as follows



Illustrate with an example. **Memorize** this result.

8. Given the natural response of a stable first order RC circuit with $a > 0$ as follows

$$y_n(t) = Ke^{-at}u(t)$$

show that the pole of the corresponding discrete system with natural response as follows

$$y_n[n] = Ke^{-anT_s}u[n] = K\left(e^{-aT_s}\right)^n u[n]$$

is located within the unit circle

9. Given the natural response of a stable second order RC circuit with $a > 0$ as follows

$$y_n(t) = Ke^{-at}\cos(bt + \theta)u(t)$$

Show that the pole of the corresponding discrete system with natural response as follows

$$y_n[n] = Ke^{-anT_s}\cos\left(bnT_s + \theta\right)u[n]$$

is located within the unit circle

10. The objective of this and the next problem is to review the relation between the frequency responses and pole-zero diagrams of some basic digital filters. Sketch the locations of the poles and zeros of a second order recursive filter (IIR filter) that is
    a. Lowpass
    b. Bandpass
    c. Highpass

11. Sketch the locations of the poles and zeros of an eighth order nonrecursive digital filter (FIR filter) that is
    a. Lowpass
    b. Bandpass
    c. Highpass