ECE 306 - POLES AND ZEROS - INVESTIGATION 24
MORE POLES AND ZEROS - PART I

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In Investigation 9 we showed how the natural responses of linear difference equations with constant coefficients are related to their characteristic roots. The objective of this and the next Investigation is to show how these results can be derived from the z-transform. We'll show in particular that the characteristic roots we worked with in Investigation 9 are the same as the poles of the transfer functions $H(z)$.

1. Find the transfer function $H(z)$ for

$$y[n] = 0.4y[n-1] - 0.5y[n-2] + x[n] + 1.3x[n-2]$$

2. Given the following transfer function $H(z)$

$$H(z) = \frac{z(z-1)}{(z+0.5)(z-0.4)}$$

with **zeros** z_k equal the roots of the numerator and **poles** p_k equal to the roots of the denominator

   a. Find the zeros of $H(z)$
   b. Find the poles of $H(z)$
   c. Find the difference equation with transfer function $H(z)$
   d. Verify that the characteristic roots are the same as the poles

3. Generalizing on the result of Problem (2) we have that transfer functions $H(z)$ can be expressed in terms of their poles (characteristic roots) and zeros as follows

$$H(z) = K\frac{(z-z_1)(z-z_2)\cdots(z-z_m)}{(z-p_1)(z-p_2)\cdots(z-p_n)}$$

where K is a constant depending on the system. Use this result to find H(z) for discrete systems with poles and zeros as follows

   a. $z_1 = 0.5, \quad z_2 = 0, \quad p_1 = -0.5 + j0.3, \quad p_2 = -0.5 - j0.3, \quad K = 2$
   b. $z_1 = 0.3 + j0.4, \quad z_2 = 0.3 - j0.4, \quad p_1 = p_2 = 0, \quad K = 3$

4. Find the poles and zeros for discrete systems with the following difference equations

   a. $y[n] = 0.5y[n-1] - 0.3y[n-2] + x[n] + 0.3x[n-1]$
   b. $y[n] = 2x[n] + x[n-1] - 0.5x[n-2]$

5. The objective of this problem is to make use of the inverse z-transform to illustrate the relationship between the poles of $H(z)$ and the natural response of the linear discrete system. Given

$$Y(z) = H(z)X(z) = \frac{z^2}{(z-0.5)(z+0.3)}X(z)$$

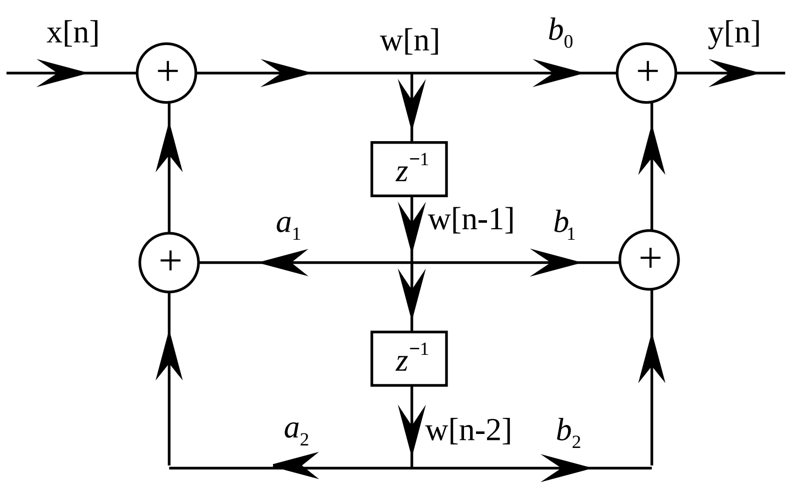
   a. Find $Y(z)$ if $x[n]$ is the unit step

b. Do a partial fraction expansion of $Y(z)$
c. Identify those terms in the partial fraction expansion corresponding to the natural response and those corresponding to the forced response
d. Find the poles of $H(z)$
e. Verify that the natural response is from the terms in the partial fraction expansion determined by the poles

6. Generalizing on the results of Problem (5) it can be shown that the natural response is a sum of terms determined by the poles of $H(z)$. Given this result, sketch the natural responses of discrete systems with poles
   a. $p_1 = 0.5$
   b. $p_1 = -0.5$
   c. $p_1 = 1$
   d. $p_1 = -1$
   e. $p_1 = -0.5 + j0.5, \quad p_1 = -0.5 - j0.5$
   f. $p_1 = 2$
   g. $p_1 = -2$

7. What are the poles of linear discrete systems with natural responses
   a. $y_n[n] = 2(0.5)^n u[n]$
   b. $y_n[n] = 2(-0.5)^n u[n]$
   c. $y_n[n] = 2(0.5)^n \cos(0.3n + 1.2)u[n]$
   d. $y_n[n] = 2u[n]$

8. Find the difference equation for $y[n]$ as a function of $x[n]$ for the following general Direct Form II realization. Hint - use the same procedure as in the last Investigation.



9. Do an example to illustrate the fact that the poles of nonrecursive digital filters are all at the origin. **Memorize** this result

10. Can nonrecursive digital filters ever be unstable. Hint - make use of your result in Problem (9)