ECE 306 - THE VERY BASICS - INVESTIGATION 2
REVIEW OF FREQUENCY RESPONSE AND SPECTRUM

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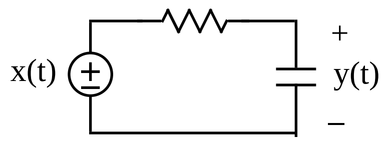
The main objective of the last Investigation was to introduce difference equations like the following

$$y[n] = 2x[n] \qquad y[n] = 0.5y[n-1] + 1.5x[n]$$

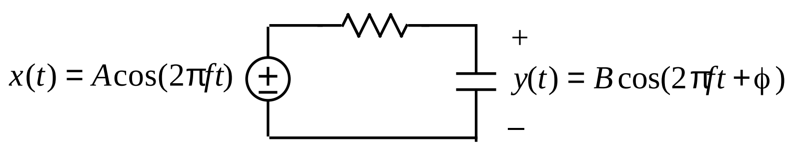
for discrete amplifiers, attenuators, filters and so on. The objective of this Investigation is to review the basics of frequency domain analysis of analog systems that we'll be needing in future Investigations.

1. From circuit analysis we know that the complete response of an RC circuit like the following



   to a sinusoid $x(t) = A\cos(2\pi ft)$ is a sinusoid of the *same* frequency f plus a transient response. And that once the transient response "decays" the only thing left at the ouput will be the sinusoid. We call this the *sinusoidal steady state response*. Sketch what the sinusoidal steady state response of this circuit looks like.

2. The objective of this problem is to see how varying the frequency f of the sinusoidal source in the following RC circuit



   affects the amplitude B of the sinusoidal steady state response

   a. Describe what's going on in the circuit - how the charges are flowing - as the sinsuoid goes back and forth from positive to negative
   b. How does increasing the frequency of the sinusoidal source affect the time the charges have to accumulate on the plates of the capacitor
   c. How does increasing the frequency of the sinusoidal source affect the amplitude B of the output
   d. Sketch B/A as a function of frequency f

3. Based on our results in Problem (2) we say that the **frequency response** of a linear circuit is how the amplitude and phase of the *sinusoidal steady state response* vary as a function of frequency. **Memorize** this definition forever. Then find the amplitude of the sinusoidal steady state response of a linear circuit to $x(t) = A\cos(2\pi 1000t)$ if

   (1) $A = 5$

   (2) $B/A = 3$ at $f = 1\,\text{Hz}$

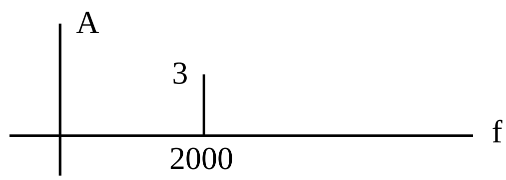
4. In this problem we review what is meant by the spectrum of a sinusoid

   a. Sketch $x(t) = 5\cos(2\pi 1000t)$ as a function of time
   b. What is the amplitude A of $x(t)$ and what is its frequency f in Hz

c. Describe the relationship between $x(t) = A\cos(2\pi ft)$ and its *single-sided spectral plot* as follows

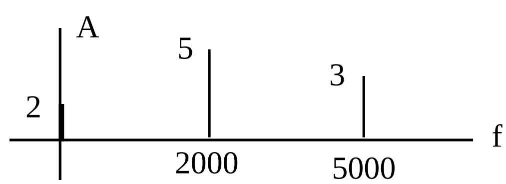
d. Draw the single-sided spectral plot of $x(t)$ in part (a)

e. Write out $x(t)$ for the sinusoid with single-sided spectral plot as follows



f. Draw a single-sided spectral plot for $x(t) = 5\cos(2\pi 1000t) + 3\cos(2\pi 2000t)$. Hint - draw a spectral line for each sinusoid of the sum.

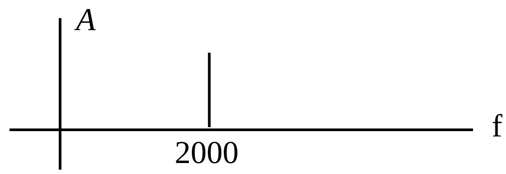
g. Write out an expression for $x(t)$ with the following single-sided spectral plot

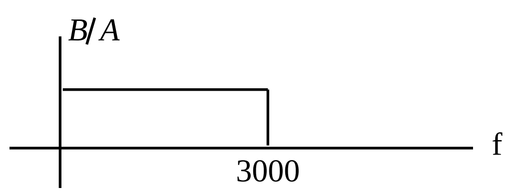


Note that a spectral line at $f = 0$ is for a constant term.

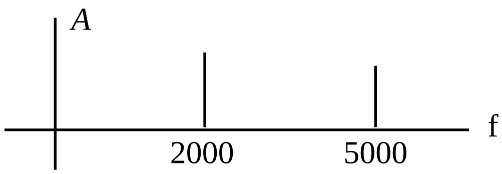
5. Sketch the spectrums B at the outputs of linear circuits with frequency responses and input spectrums as follows

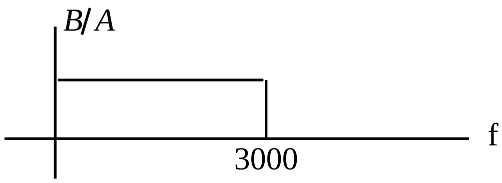
a.



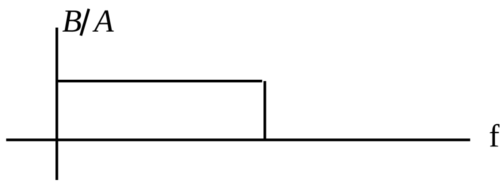


b.





6. Why do we call linear circuits with frequency responses like the following low pass filters



7. The *bandwidth* of a sum of sinusoids like $x(t) = 5\cos(2\pi 100t) + 3\cos(2\pi 200t) + \cos(2\pi 300t)$ is the highest frequency. Therefore the bandwidth of $x(t)$ is $BW = 300Hz$
   a. What is the bandwidth of $x(t) = 3\cos(2\pi 2000t) + \cos(2\pi 4000t)$
   b. Write out and sketch the single-sided spectral plot of a sum of sinusoids $x(t)$ with bandwidth $BW = 7000Hz$

8. A signal $x(t)$ with spectrum B/A is **bandlimited** by f_b if

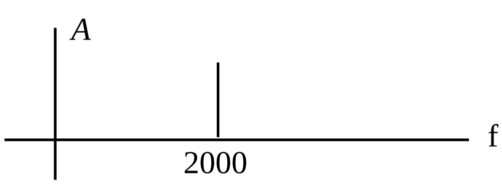
$$X(f) = 0 \quad \text{for all } f \text{ such that } |f| > f_b$$

   a. Sketch the spectrum B/A of a signal $x(t)$ that is bandlimited by f_b = 2000 Hz
   b. What is f_b for $x(t) = 3 + 2\cos(2\pi 100t) + 4\cos(2\pi 300t)$

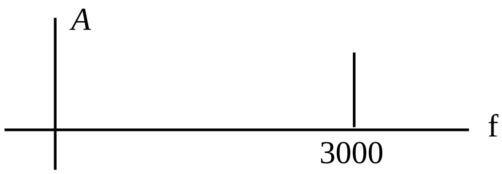
9. Which of the following signals are bandlimited by f_b = 2000 Hz

a.

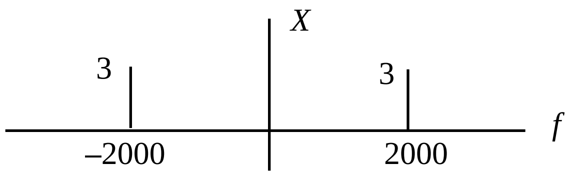
b.



c.



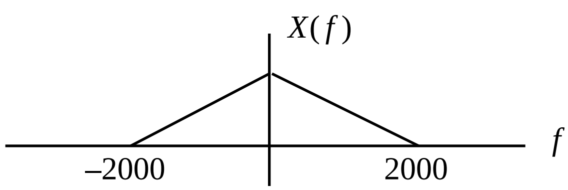
10. Up to now all our spectral plots have been single-sided. But double-sided spectral plots like the following



for $x(t) = 6\cos(2\pi 2000t)$ are easier to work with when we calculate the spectrums of nonperiodic signals like single pulses

a. Explain how double-sided spectral plots are different from single-sided spectral plots
b. Sketch the double-sided spectral plot of $x(t) = 10\cos(2\pi 4000t)$

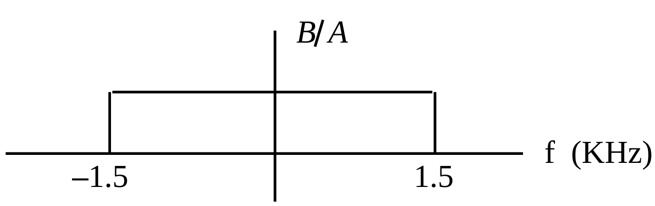
11. Generalizing on our previous results we can show that nonperiodic signals $x(t)$ like single pulses also have spectrums $X(f)$ - now referred to as spectral densities - very much like our spectral plots except they're continuous like the following



We call $X(f)$ the *Fourier Transform* of $x(t)$. As before we have that if $y(t)$ is the response of an analog circuit to $x(t)$ then their spectrums $Y(f)$ and $X(f)$ are related as follows

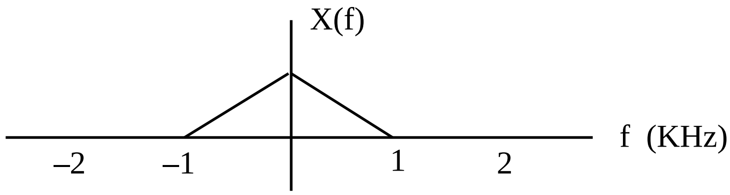
$$Y(f) = (\text{Frequency Response of the circuit})\ X(f)$$

Make use of this result to find the spectrum of the output of a linear circuit with the following frequency response

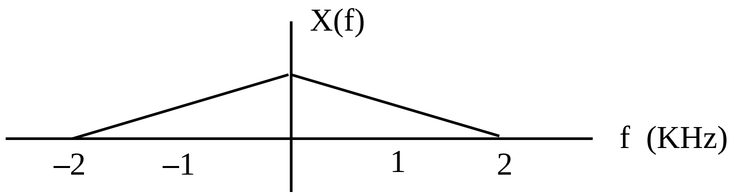


when the inputs have the spectrums

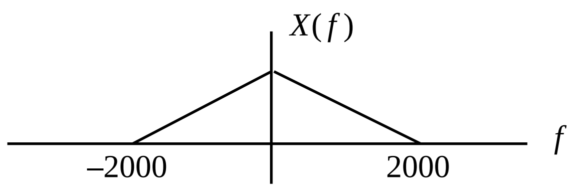
a.



b.



12. Given a signal with the following spectrum



   a. How can you tell that this signal is bandlimited
   b. What is the bandwidth of $X(f)$

13. Math Review - sketch

$$Y(f) = \sum_{k=-1}^{1} X(f - 5k)$$

if $X(f)$ has the following spectrum