ECE 306 - FOURIER ANALYSIS - INVESTIGATION 19
INTRODUCTION TO THE DFT

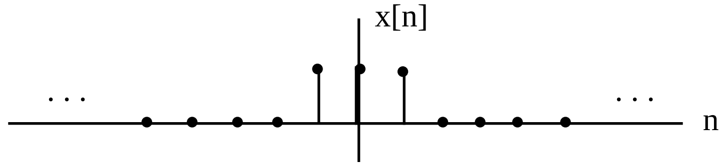
WINTER 2007 **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

In the last Investigations we introduced and worked with the following equation

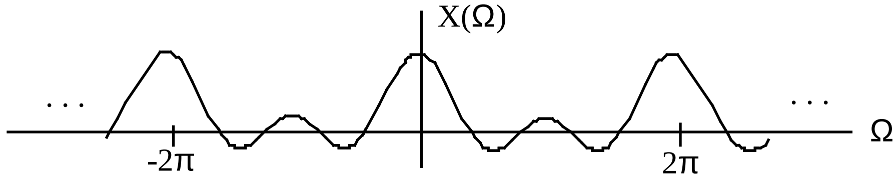
$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\Omega}$$

for Discrete Time Fourier Transforms - the continuous spectral densities of nonperiodic sequences like the following pulse



We made a point to emphasize that X(Ω) is

(1) A *continuous* function of the variable Ω
(2) *Periodic* of period 2π like the following example



Now Discrete Time Fourier Transforms of sequences are great. But in general we can't calculate them because they're **infinite** sums calculated at an **infinite** number of frequencies.

The objective of this Investigation is to define the *Discrete Fourier Transform (DFT)* which, as we'll see, is basically the same as the Discrete Time Fourier Transform except that it's a **finite** sum calculated at a **finite** number of frequencies. The DFT is the workshorse of discrete Fourier analysis.

We will restrict ourselves in this introductory Investigation to *finite duration* discrete sequences like the above pulse $x[n]$.

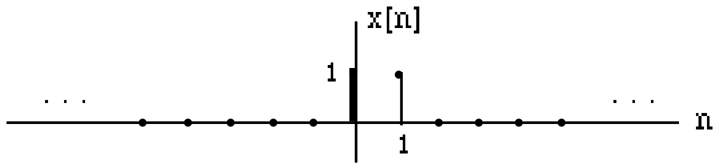
1. The **Discrete Fourier Transform (DFT)** of a finite sequence $x[n]$ is by definition

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-jk(2\pi/N)n}$$

where $\mathbf{X}[k]$ is calculated for k = 0, 1, 2, . . . , $N-1$. Note that this equation is exactly the same as that for the DTFS coefficients of a periodic sequence $x[n]$ of period N except it doesn't have the factor $1/N$. As a result we can retrieve the $x[n]$'s with the inverse DFT (IDFT) as follows

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{jk(2\pi/N)n}$$

a. Calculate and plot the magnitude and the phase of the DFT of the following sequence for $N = 4$



b. Now calculate the IDFT of your result in part (a) and verify that you get $x[n]$ back

2. Since the DFT is of the same form as the DTFS it is also periodic of period N. **Memorize** this result. Then make use of it to sketch the DFT in Problem (1) from $k = -4$ to $k = 8$.

3. The objective of this problem is to confirm that the DFT has the same basic properties as the DTFS.
   a. Show that the DFT is linear - that

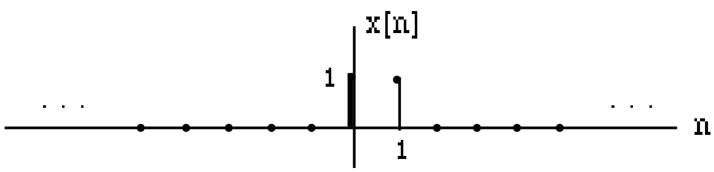
$$DFT\left[a_1 x_1[n] + a_2 x_2[n]\right] = a_1 X_1[k] + a_2 X_2[k]$$

   b. Show that $\mathbf{X}[-k] = \mathbf{X}\ [k]$ for real sequences $x[n]$

4. The objective of this problem is to demonstrate how the DFT of a *finite duration* sequence $x[n]$ is related to its Discrete Time Fourier Transform as follows

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\Omega}$$

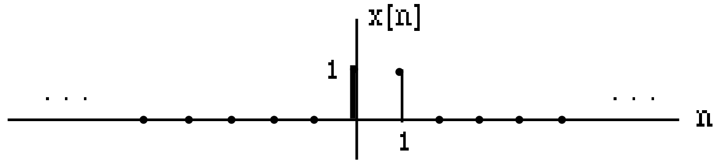
For the following sequence $x[n]$



   a. Calculate the Discrete Time Fourier Transform $X(\Omega)$ and then plot its magnitude
   b. Calculate and plot the DFT for $N = 4$
   c. Verify that the DFT is equal to the Discrete Time Fourier Transform at the corresponding values of $\Omega = k(2\pi/N)$
   d. Draw the DFT values on a graph of $X(\Omega)$

5. Generalize on the results of Problem (4) and show that if $x[n]$ is equal to zero for all n outside the range $0 \le n \le N-1$, then the values of the DFT are always equal to the Discrete Time Fourier Transform at the corresponding frequencies.

6. The objective of this problem is to see what happens when we increase the value of N when we calculate the DFT of a sequence like the following



a. Calculate the DFT for N = 4. Then plot the $X[k]$'s on a plot of the DTFT $X(\Omega)$
b. Repeat part (a) for N = 8
c. Describe what's happening to the resolution as N increases. This is called *zero padding*