ECE 306 - FOURIER ANALYSIS - INVESTIGATION 16
DISCRETE TIME FOURIER SERIES - PART II

WINTER 2007 **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

We demonstrated in the last Investigation that if $x[n]$ is a periodic sequence of period N then it can be expressed as a finite sum of discrete complex exponentials as follows

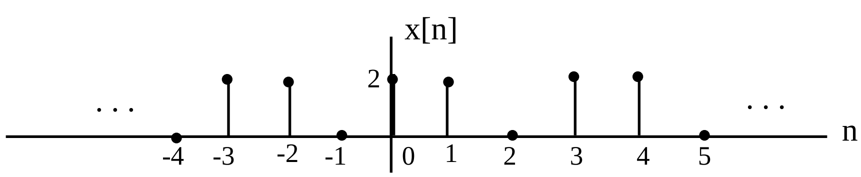
$$x[n] = \sum_{k=0}^{N-1} X_k e^{jk(2\pi/N)n}$$

with Discrete Time Fourier Series Coefficients $\mathbf{X}_k$'s calculated from the following finite sums

$$\mathbf{X}_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk(2\pi/N)n}$$

The main objective of this Investigation is to develop some of the basic properties of Discrete Time Fourier Series and show how Discrete Time Fourier Series (DTFS) expansions can be used to calculate steady state responses.

1. The objective of this is to practice calculating some DTFS coefficients $\mathbf{X}_k$. Given the following periodic sequence



   a. What is the period N
   b. Find and plot the magnitudes $|X_k|$ and phases X_k of the DTFS coefficients $\mathbf{X}_k$ on separate graphs for $k = 0, \ldots, N-1$
   c. Make use of your result in part (b) to express $x[n]$ as a sum of complex exponentials
   d. And finally make use of your result in part (c) to verify that it gives the correct values of $x[0]$, $x[1]$ and $x[2]$

2. The objective of this and the next several problems is to develop some of the basic properties of the DTFS. The objective of this problem in particular is to demonstrate that the Discrete Fourier coefficients $\mathbf{X}_k$ are periodic with period N just like the $x[n]$'s

   a. Make use of the equation for the $\mathbf{X}_k$'s as follows

$$\mathbf{X}_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk(2\pi/N)n}$$

   to show that they are periodic with period N - that they satisfy the following expression

$$\mathbf{X}_{k+N} = \mathbf{X}_k$$

Memorize this result.

b. Find and plot the magnitudes $|X_k|$ and phases X_k of the DTFS coefficients $\mathbf{X}_k$ from Problem (1) on separate graphs for $-2N \le k \le 2N$

c. What do you observe to be the relationship between $\mathbf{X}_{-k}$ and $\mathbf{X}_k$ in part (b)

3. Generalizing on the result of Problem (2c) it can be shown that if x[n] is a periodic sequence of real valued samples then

$$\mathbf{X}_{-k} = \mathbf{X}_k^*$$

Memorize this result. Then use it to calculate the following DTFS coefficients for a periodic sequence of real numbers of period $N = 5$ with the following coefficients

$$\mathbf{X}_0 = 1 \qquad \mathbf{X}_1 = 2e^{j1.2} \qquad \mathbf{X}_2 = 3e^{-j0.5}$$

a. Find $\mathbf{X}_{-1}$ and $\mathbf{X}_{-2}$

b. And then find $\mathbf{X}_3$ and $\mathbf{X}_4$. Hint - make use of the fact that the DTFS is periodic with period $N = 5$

c. What do your results imply about the relationship between $\mathbf{X}_1$ and $\mathbf{X}_4$ and between $\mathbf{X}_2$ and $\mathbf{X}_3$ when $N = 5$ like it is in this problem.

4. Generalizing on Problem (3) we have for periodic sequences of real numbers that

$$\mathbf{X}_1 = \mathbf{X}_{N-1}\,,\ \mathbf{X}_2 = \mathbf{X}_{N-2}\,,\cdots$$

Verify that this is true for a periodic sequence x[n] of your own.

5. Up to now we've been calculating the $\mathbf{X}_k$'s as a sum from $n = 0$ to $n = N-1$ as follows

$$\mathbf{X}_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk(2\pi/N)n}$$

But since $x[n]$ is periodic we can start the sum at any value of n as long as we take it over N consecutive values. Verify that you get the same values for the $\mathbf{X}_k$'s of Problem (1) from the foillowing sum. **Memorize** this result.

$$\mathbf{X}_k = \frac{1}{N}\sum_{n=-1}^{N-2} x[n]e^{-jk(2\pi/N)n}$$

6. Show that the DTFS is linear. In particular, show that

$$\text{if} \quad x[n] = a_1x_1[n] + a_2x_2[n] \quad \text{then} \quad \mathbf{X}_k = a_1\mathbf{X}_{1k} + a_2\mathbf{X}_{2k}$$

for any two sequences of period N