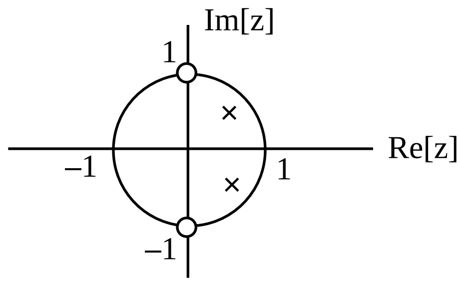
ECE 306 - FREQUENCY RESPONSE - INVESTIGATION 14
FREQUENCY RESPONSES OF DISCRETE SYSTEMS - PART V

WINTER 2007 **A.P. FELZER**

In the last Investigation we introduced pole-zero diagrams like the following



and showed how the locations of the poles and zeros are related to the frequency response. The main objective of this Investigation is to show how the poles of a difference equation are related to its characteristic roots and therefore its natural response as discussed in Investigation 8

1. We begin with some review problems. Sketch the pole-zero diagrams of 2nd order difference equations with two poles and two zeros with frequency responses that are
   a. Lowpass
   b. Bandpass
   c. Highpass
   d. Bandstop

2. Given the following difference equation

$$y[n] = 0.4y[n-1] - 0.6y[n-2] + 2x[n] - x[n-1]$$

   Find the
   a. Frequency Response $H\left(e^{j2\pi fT_s}\right)$
   b. Transfer Function $H(z)$ from $H\left(e^{j2\pi fT_s}\right)$
   c. Poles and zeros of $H(z)$

3. Up to now we've been obtaining $H(z)$ from $H\left(e^{j2\pi fT_s}\right)$ by replacing every $e^{j2\pi fT_s}$ by z. But by careful observation of our results in Problem (2) and in the previous Investigation we see that we can obtain $H(z)$ directly from a difference equation by replacing

$$y[n] \rightarrow Y(z), \quad y[n-1] \rightarrow z^{-1}Y(z), \quad y[n-2] \rightarrow z^{-2}Y(z) \ \ldots$$
$$x[n] \rightarrow X(z), \quad x[n-1] \rightarrow z^{-1}X(z), \quad x[n-2] \rightarrow z^{-2}X(z) \ \ldots$$

   and then solve for

$$H(z) = \frac{Y(z)}{X(z)}$$

   Memorize this observation. Verify that this algorithm works for the difference equation of Problem (2)

4. The objective of this problem is to show that the characteristic roots we calculated in

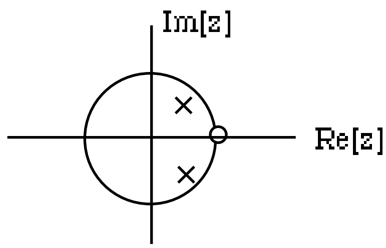
Investigation 8 for finding the natural response of a difference equation are exactly the same as the poles of its transfer function $H(z)$. Given the following difference equation

$$y[n]-0.4y[n-1]+0.8y[n-2]=x[n]$$

a. Find the characteristic roots for calculating the natural response as introduced in Investigation 5
b. Find the poles of $H(z)$
c. Verify that the poles are equal to the characteristic roots
d. Explain why the characteristic roots are equal to the poles. Hint - compare the algorithms for obtaining the characteristic roots and for obtaining $H(z)$

5. Given a transfer function $H(z)$ with zero $z_1=-1$ and poles $p_1, p_1=-0.6\pm j+0.5$
    a. Sketch the pole-zero diagram
    b. Sketch the frequency response
    b. Find the natural response
    c. Sketch the natural response

6. Given a pole-zero diagram like the following



How does moving the poles closer to the unit circle affect the
    a. Frequency response. Draw graphs to illustrate
    b. Natural response

7. Find the sinusoidal steady state response to

$$y[n]=0.6y[n-1]-0.2y[n-2]+2x[n]-x[n-1]$$

to $x(t)=3\cos(2\pi 1000t+1.2)$ sampled at $f_s=2500$ samples/sec. Hint - First find $H(z)$ and then make use of it to obtain $H\left(e^{j2\pi fT_s}\right)$. **Memorize** this technique for finding sinusoidal steady state responses of difference equations