ECE 306L - DIGITAL FILTERS - LAB 7
POLES, ZEROS AND FREQUENCY RESPONSE

SPRING 2007 **FELZER/KANG**

OBJECTIVE

The objective of this lab is to see how the frequency response of a digital filter is related to the locations of its poles and zeros.

PRELAB

1. Given $z_1 = 2 + j3$
   a. Use your calculator to find

      $|z_1| = |2 + j3|$ = Distance in the complex plane between $2 + j3$ and the origin

   b. Use Matlab to find $|z_1| = |2 + j3|$
   c. Verify that your results in parts (a) and (b) are the same

2. Given $z_1 = 2 + j3$ and $z_2 = -1 + j2$
   a. Use your calculator to find

      $|z_1 - z_2|$ = Distance in the complex plane between z_1 and z_2

   b. Use Matlab to find $|z_1 - z_2|$
   c. Verify that your results in parts (a) and (b) are the same

3. Given $z_1 = e^{j1.2}$ and $z_2 = -1 + j2$
   a. Use your calculator to find $|z_1 - z_2|$
   b. Use Matlab to find $|z_1 - z_2|$
   c. Verify that your results in parts (a) and (b) are the same

4. Given the complex number $z = e^{jb}$
   a. Where are the points z in the complex plane. Draw a picture to illustrate
   b. What happens to z as b increases

5. Find the points $e^{j2\pi f / f_s}$ in the complex plane when
   a. $f = 0$
   b. $f = f_s/4$
   c. $f = f_s/2$

6. Given the following transfer function

$$H(z) = 2\frac{(z-1)(z+1)}{\left(z - 0.8e^{j\pi/2}\right)\left(z - 0.8e^{-j\pi/2}\right)}$$

   a. What are the poles and zeros
   b. Sketch the pole-zero diagram. Be sure to draw the unit circle

c. Why do we draw the unit circle
d. Make use of the pole-zero diagram to sketch the frequency response for $-f_s/2 \le f \le f_s/2$
e. Is this filter lowpass, highpass or bandpass. How can you tell

7. Draw the pole-zero diagram of a
   a. 1st order lowpass recursive digital filter
   b. 2nd order lowpass recursive digital filter

8. Draw the pole-zero diagram of a
   a. 1st order highpass recursive digital filter
   b. 2nd order highpass recursive digital filter

LAB

1. Write a Matlab program to plot the following distance in the complex plane

$$d(f) = \left| e^{j2\pi f/f_s} - 0.8 \right|$$

   as a function of f for $-f_s/2 \le f \le f_s/2$ for $f_s = 10^4$ samples/sec

2. Given the following first order lowpass recursive digital filter with transfer function

$$H(z) = b_0 \frac{z+1}{z-p_1}$$

   a. Write a Matlab function for plotting the frequency response $\left| H\left(e^{j2\pi f/f_s}\right) \right|$ as a function of f_s, b_0 and p_1
   b. Run your program in part (a) for $b_0 = 2$ and $p_1 = 0.8$ in the frequency range $-f_s/2 \le f \le f_s/2$ for $f_s = 5000$ samples/sec
   c. What does your graph predict for the sinusoidal steady state response to $x(t) = 5\cos(2\pi 1000t)$ sampled at $f_s = 5000$ samples/sec
   d. Verify your result in part (c) with a Simulink realization

3. Given the following first order highpass recursive digital filter with transfer function

$$H(z) = b_0 \frac{z-1}{z-p_1}$$

   a. Write a Matlab function for plotting the frequency response $\left| H\left(e^{j2\pi f/f_s}\right) \right|$ as a function of f_s, b_0 and p_1 for $f_s = 5000$ samples/sec
   b. Run your program in part (a) for $b_0 = 2$ and $p_1 = -0.8$
   c. What does your graph predict for $y(t)$ when $x(t) = 5\cos(2\pi 1000t)$
   d. Verify your result in part (c) with a Simulink realization

4. Run your programs in Problems (2) and (3) with values of b_0 so that
   a. The gain of the lowpass is one at $f = 0$
   b. The gain of the highpass is one at $f = f_s/2$

5. Run your programs in Problems (2) and (3) to find out what happens to

a. The frequency response of the first order lowpass as the pole moves closer to +1. Draw several graphs on the same plot
b. The frequency response of the first order highpass as the pole moves closer to -1. Draw several graphs on the same plot

6. Make use of Matlab to plot the frequency response of a 2nd order bandpass recursive digital filter as follows

$$H(z) = b_0 \frac{(z+1)(z-1)}{(z-p_1)(z-p_2)}$$

for various locations of the poles to see how the center frequency and the sharpness of the response is affected. In each case choose b_0 so that the maximum gain of the filter is equal to one

POSTLAB

1. How does the location of the pole of a 1st order lowpass affect the sharpness of the frequency response

2. How does the location of the pole of a 1st order highpass affect the sharpness of the frequency response

3. How does the location of the poles of a 2nd order bandpass affect the location of the center frequency and the sharpness of the frequency response