ECE 306L - DIGITAL FILTERS - LAB 6
SIMPLE RECURSIVE DIGITAL FILTERS

SPRING 2007 **FELZER/KANG**

OBJECTIVE

The objective of this lab is to use Simulink to obtain the frequency responses of some simple recursive digital filters.

PRELAB

1. Given a recursive digital filter with the following difference equation

$$y[n] = 0.9y[n-1] + 0.05x[n] + 0.05x[n-1]$$

   a. Find $H(z) = \frac{Y(z)}{X(z)}$
   b. Find the frequency response

$$H\left(e^{j2\pi f/f_s}\right) = H(z)\Big|_{z=e^{j2\pi f/f_s}}$$

   c. Use $H\left(e^{j2\pi f/f_s}\right)$ to obtain an equation for the sinusoidal steady state response $y[n]$ when $x(t) = 5\cos(2\pi 2000t)$ is sampled at $f_s = 10^4$ samples/sec
   d. Use Matlab to calculate $\left|H\left(e^{j2\pi f/f_s}\right)\right|$ at ten representative frequencies for when $f_s = 10^4$ samples/sec
   e. Use Matlab to obtain a full page plot of the magnitude of the frequency response as follows

$$\left|H\left(e^{j2\pi f/f_s}\right)\right| \quad \text{for} \quad -f_s/2 \le f \le f_s/2$$

   for $f_s = 10^4$ samples/sec
   f. Verify that your calculations in part (d) agree with your graph in part (e)
   g. Is this filter lowpass, highpass or bandpass. How can you tell
   h. Draw a Simulink block diagram of delays, multipliers and adders for realizing the difference equation for when $x[n]$ is a sinusoid

2. Given a digital filter with the following transfer function

$$H(z) = \frac{Y(z)}{X(z)} = b_o \frac{z-1}{z+0.9}$$

   a. Find the frequency response

$$H\left(e^{j2\pi f/f_s}\right) = H(z)\Big|_{z=e^{j2\pi f/f_s}}$$

   b. Find b_0 so the gain $H\left(e^{j2\pi f/f_s}\right)$ is equal to one when $f = f_s/2$
   c. Use $H\left(e^{j2\pi f/f_s}\right)$ to obtain an equation for the sinusoidal steady state response $y[n]$ when $x(t) = 5\cos(2\pi 2000t)$ sampled at $f_s = 10^4$ samples/sec

d. Use Matlab to calculate $\left|H\left(e^{j2\pi f/f_s}\right)\right|$ at ten representative frequencies for when $f_s = 10^4$ samples/sec
e. Use Matlab to obtain a full page plot of the magnitude of the frequency response as follows

$$\left|H\left(e^{j2\pi f/f_s}\right)\right| \quad \text{for} \quad -f_s/2 \le f \le f_s/2$$

for $f_s = 10^4$ samples/sec
f. Verify that your calculations in part (d) agree with your graph in part (e)
g. Is this filter lowpass, highpass or bandpass. How can you tell
h. Find the difference equation for this filter
i. Draw a Simulink block diagram of delays, multipliers and adders for realizing the difference equation for when $x[n]$ is a sinusoid

3. Given a digital filter with the following transfer function

$$H(z) = \frac{Y(z)}{X(z)} = b_o \frac{(z+1)(z-1)}{(z+j0.9)(z-j0.9)}$$

a. Find the frequency response

$$H\left(e^{j2\pi f/f_s}\right) = H(z)\big|_{z=e^{j2\pi f/f_s}}$$

b. Find b_0 so the gain is equal to one when $f = f_s/4$
c. Use $H\left(e^{j2\pi f/f_s}\right)$ to obtain an equation for the sinusoidal steady state response $y[n]$ when $x(t) = 5\cos(2\pi 2000t)$ sampled at $f_s = 10^4$ samples/sec
d. Use Matlab to calculate $\left|H\left(e^{j2\pi f/f_s}\right)\right|$ at ten representative frequencies for when $f_s = 10^4$ samples/sec
e. Use Matlab to obtain a full page plot of the magnitude of the frequency response as follows

$$\left|H\left(e^{j2\pi f/f_s}\right)\right| \quad \text{for} \quad -f_s/2 \le f \le f_s/2$$

for $f_s = 10^4$ samples/sec
f. Verify that your calculations in part (d) agree with your graph in part (e)
g. Is this filter lowpass, highpass or bandpass. How can you tell
h. Find the difference equation for this filter
i. Draw a Simulink block diagram of delays, multipliers and adders for realizing the difference equation for $y[n]$ when $x[n]$ is a sinusoid

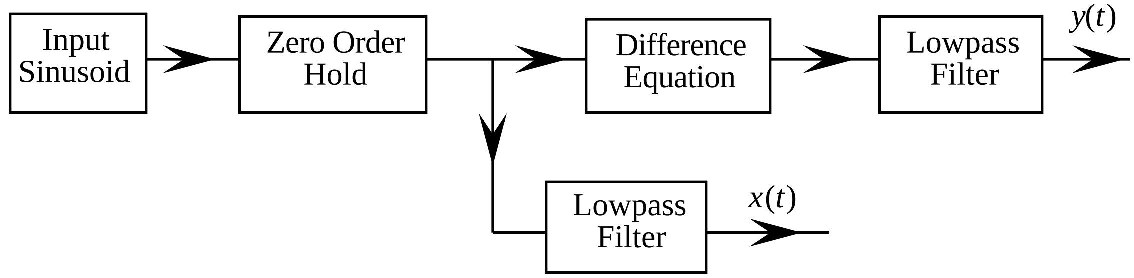
LAB

1. For the difference equation of Problem (1) as follows

$$y[n] = 0.9y[n-1] + 0.05x[n] + 0.05x[n-1]$$

with $f_s = 10^4$ samples/sec

a. Use Simulink to obtain the sinusoidal steady state responses at the frequencies you chose in Prelab Problem (1d). Put *both* $x_{S/H}(t)$ and $y_{S/H}(t)$ through lowpass analog filters as follows



to obtain the sinusoids for calculating the gains in part (b) below. Make screen captures at low, medium and high frequencies of both $x(t)$ and $y(t)$. Be sure to set the sample time of the zero order hold

b. Use your Simulink results to calculate $\left|H\left(e^{j2\pi f/f_s}\right)\right|$ for each of your sinusoids. Put your results in a Table

c. Put your values of $\left|H\left(e^{j2\pi f/f_s}\right)\right|$ on your Matlab graph from the prelab

2. For the difference equation of Problem (2) with $f_s = 10^4$ samples/sec
   a. Use Simulink to obtain the sinusoidal steady state responses at the frequencies you chose in Prelab Problem (2d). Put both $x_{S/H}(t)$ and $y_{S/H}(t)$ through analog lowpass filters to obtain the corresponding sinusoids. Make screen captures at low, medium and high frequencies
   b. Use your Simulink results to calculate $\left|H\left(e^{j2\pi f/f_s}\right)\right|$ for each of your sinusoids. Put your results in a Table
   c. Put your values of $\left|H\left(e^{j2\pi f/f_s}\right)\right|$ on your Matlab graph from the prelab

3. For the difference equation of Problem (3) with $f_s = 10^4$ samples/sec
   a. Use Simulink to obtain the sinusoidal steady state responses at the frequencies you chose in Prelab Problem (3d). Put both $x_{S/H}(t)$ and $y_{S/H}(t)$ through analog lowpass filters to obtain the corresponding sinusoids. Make screen captures at low, medium and high frequencies
   b. Use your Simulink results to calculate $\left|H\left(e^{j2\pi f/f_s}\right)\right|$ for each of your sinusoids. Put your results in a Table
   c. Put your values of $\left|H\left(e^{j2\pi f/f_s}\right)\right|$ on your Matlab graph from the prelab

POSTLAB

1. Compare your Matlab and Simulink results for $\left|H\left(e^{j2\pi f/f_s}\right)\right|$ for each of the three filters

2. Given the following two sinusoids

$$x_1(t) = 5\cos(2\pi f t) \quad \text{and} \quad x_2(t) = 5\cos\left(2\pi(f + f_s)t\right)$$

a. Describe how $x_1(t)$ and $x_2(t)$ are related
b. Show that the samples of $x_1(t)$ and $x_2(t)$ are the same with $x_1[n] = x_2[n]$

3. Make use of Problem (2) to explain why $\left|H\left(e^{j2\pi f/f_s}\right)\right|$ is periodic with period f_s

4. Sketch the frequency response $\left|H\left(e^{j2\pi f/f_s}\right)\right|$ for each of the following filters for $-2f_s \leq f \leq 2f_s$

   a. Lowpass
   b. Highpass
   c. Bandpass