ECE 306L - TRANSITION TO DISCRETE - LAB 5
THE SPECTRUMS OF SAMPLED SIGNALS

SPRING 2007 **FELZER/KANG**

OBJECTIVE

The objective of this lab is to see how fast signals must be sampled to avoid aliasing.

PRELAB

1. What is the sampling frequency f_s when the time between samples is $T_s = 0.1\,\text{msec}$

2. What is the time T_s between samples when the sampling frequency is $f_s = 2\times 10^4$ samples/sec

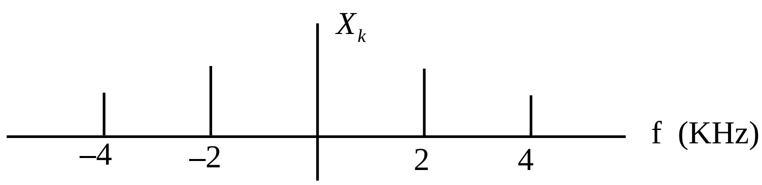
3. How does increasing the sampling frequency f_s affect the sample time T_s

4. What does the Sampling Theorem say

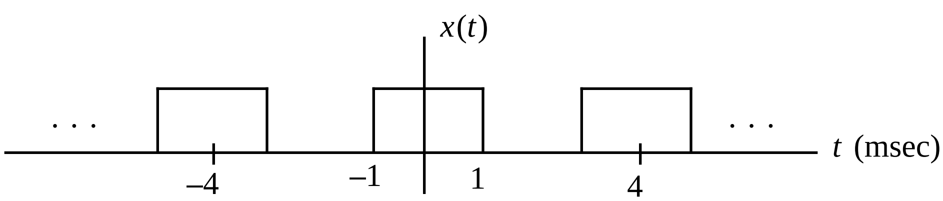
5. What is aliasing

6. How fast do we need to sample the following signals to avoid aliasing
   a. $x_1(t) = \cos(2\pi 3000t)$
   b. $x_2(t) = \cos(2\pi 3000t) + 2\cos(2\pi 4000t)$

7. How fast do we have to sample $x(t)$ with the following double-sided spectral plot to avoid aliasing



8. Given a pulse train as follows



   a. Why can $x(t)$ never be sampled fast enough to completely eliminate aliasing
   b. How fast would you sample $x(t)$ in the real world. Why

9. Use Matlab to verify that the first five samples of the following two signals are equal

$$x_1(t) = 5\cos(2\pi ft) \qquad x_2(t) = 5\cos\big(2\pi(f + f_s)t\big)$$

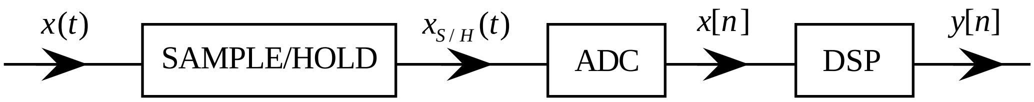
   when $f = 2000\,\text{Hz}$ and $f_s = 5000$ samples/sec. Put the results in a Matlab Table

10. Show that the two signals in Problem (9) as follows will always have the same samples

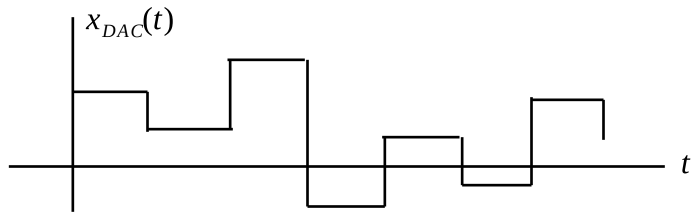
$$x_1(t) = 5\cos(2\pi f t) \qquad x_2(t) = 5\cos\left(2\pi(f + f_s)t\right)$$

Illustrate with a Matlab graph of the two functions. Draw dots at the sample times

11. In a real digital signal processing system as follows



(1) The sample-and-hold converts the continuous signal $x(t)$ to a signal $x_{S/H}(t)$ like the following



(2) The ADC converts the $x_{S/H}(t)$ signal to a binary signal $x[n]$

(3) The digital signal processor DSP processes $x[n]$ to produce $y[n]$

a. What is the purpose of the sample-and-hold
b. Sketch $x_{S/H}(t)$ for $x_1(t) = 5\cos(2\pi 1000 t)$ with $f_s = 7000$ samples/sec
c. Use Matlab to graph $x_{S/H}(t)$ in part (b)
d. Use Matlab to graph $x_{S/H}(t)$ $x_2(t) = 5\cos\left(2\pi(1000 + f_s)t\right)$
e. Verify that your graphs in parts (c) and (d) are the same

12. Use Simulink to obtain a graph of $x_{S/H}(t)$ for $x_1(t) = 5\cos(2\pi 1000 t)$ in Problem (11)

13. Make use of the fact that the two-sided spectrum of a sinusoid $x(t) = A\cos(2\pi f_x t)$ sampled at f_s samples/sec has spectral lines at the frequencies

$$\pm f_x, \quad \pm(f_s - f_x), \quad \pm(f_s + f_x), \quad \pm(2f_s - f_x), \quad \pm(2f_s + f_x), \quad \ldots$$

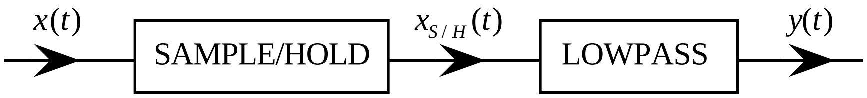
with values as follows

$$\frac{A}{2}\text{sinc}(f_x T_s), \quad \frac{A}{2}\text{sinc}((f_s - f_x)T_s), \quad \frac{A}{2}\text{sinc}((f_s + f_x)T_s), \quad \ldots$$

to sketch the double-sided spectral plots of $x_{S/H}(t)$ of $x(t) = 5\cos(2\pi 1000 t)$ at the sampling frequencies

a. $f_s = 3000$ samples/sec
b. $f_s = 5000$ samples/sec
c. $f_s = 10{,}000$ samples/sec
d. Then describe what's happening to the spectrums as the sampling frequency increases

14. The objective of this problem is to make use of the spectrums calculated in Problem (13) to see how varying the sampling frequency affects the amplitude of a reconstructed sinusoid at the output of an ideal lowpass filter with cutoff frequency $f_c = f_s/2$ as follows



In particular find and sketch $y(t)$ for each of the following sampling frequencies when $x(t) = 5\cos(2\pi 1000t)$ just like it was in the previous problem

a. $f_s = 10{,}000$ samples/sec
b. $f_s = 5000$ samples/sec
c. $f_s = 3000$ samples/sec
d. How does decreasing the sampling frequency affect the amplitude of the reconstructed sinusoid
e. How does decreasing the sampling frequency affect the ability of a real low pass filter to just pass the sinusoid at the frequency f_x

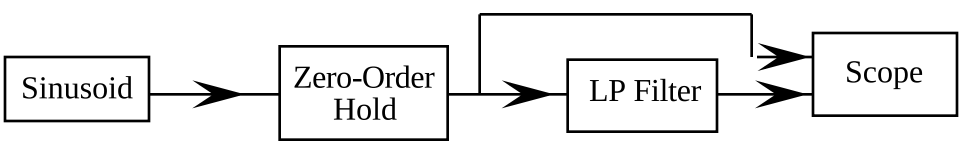
LAB

1. Obtain screen captures of $x_{S/H}(t)$ at the output of your LF398N sample/hold circuit when it's sampling $x(t) = 5\cos(2\pi 1000t)$ at the sampling frequencies
   a. $f_s = 3000$ samples/sec
   b. $f_s = 5000$ samples/sec
   c. $f_s = 10{,}000$ samples/sec

2. Obtain screen captures of the spectral plots of your sampled signals in Problem (1). Measure the frequencies and amplitudes of the spectral lines. Put your results in a Table

3. Build a lowpass filter with cutoff frequency $f_c = f_s/2$ to recover $x(t)$ when $f_s = 10{,}000$ samples/sec. Save a screen capture

4. Use Simulink to sample and recover $x(t) = 5\cos(2\pi 1000t)$ for the following sampling frequencies with a circuit as follows



   Use a 2nd order lowpass Butterworth filter from the Communications Block with a cutoff frequency of $f_c = 0.5 f_s$ Hz. Note that a line can be connected to an existing line by pressing the control key as you drag with the mouse
   a. $f_s = 3000$ samples/sec
   b. $f_s = 5000$ samples/sec
   c. $f_s = 10{,}000$ samples/sec

5. Use Simulink to "recover" $x(t) = 5\cos(2\pi 2000t)$ from its samples when $f_s = 3000$ samples/sec. Again use a 2nd order lowpass Butterworth filter from the Communications Block with a cutoff frequency of $f_c = 0.5 f_s$ Hz. What happened

POSTLAB

1. Describe how you got your sample/hold chip working

2. Use Matlab to draw double-sided spectral plots of the spectrums you measured in the lab

3. Compare the measured and calculated spectrums of your sinusoids after they pass through the sample/hold

4. Use frequency domain arguments to explain why the smaller we make the sampling frequency f_s the higher we must make the order of the lowpass filter we need to recover the sampled sinusoid. Illustrate with drawings of the spectrums