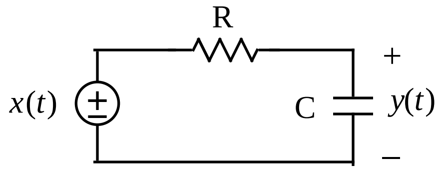
ECE 306L - FOURIER SERIES - LAB 4
HOW FILTERS AFFECT THE SPECTRUMS OF PULSE TRAINS

SPRING 2007 **FELZER/KANG**

OBJECTIVE

The objective of this lab is to see how the output $y(t)$ of a lowpass filter like the following

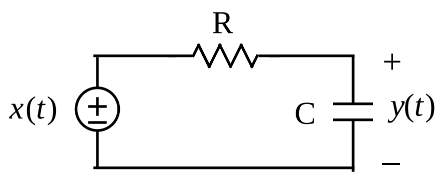


depends on both the frequency response of the filter and spectrum of the input when $x(t)$ is a pulse train

PRELAB

1. What do we mean by the frequency response of a linear circuit

2. The objective of this problem is to find the transfer function $G(j2\pi f)$ of our first order RC circuit as follows

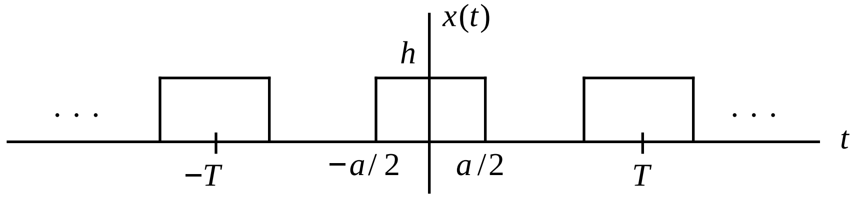


   a. Draw the phasor circuit
   b. Analyze your phasor circuit to show that its transfer function is equal to

$$G(j2\pi f) = \frac{1}{1 + j2\pi fRC}$$

3. Make use of Matlab to obtain a *full page* plot of the magnitude of the transfer function $G(j2\pi f)$ with $|G(j2\pi f)|$ plotted on a **linear scale** and f plotted on a **log scale**

4. Now suppose the input $x(t)$ of our RC circuit with $R = 10K$ is a 1/2 duty cycle pulse train as follows



   of amplitude $h = 2$ and period $T = 1\text{msec}$

   a. Find C so that the RC time constant τ is related to the pulse width a by $a = 5\tau$
   b. Sketch what you expect the circuit's response to the pulse train looks like

c. Use time domain arguments involving τ to explain your graph in part (b)
d. Use Matlab to plot the magnitude of the envelope $X_{env}(f)$ with a dashed line
e. Draw in the spectral lines by hand
f. Use Matlab to plot the magnitude of the envelope $Y_{env}(f)$ as follows with a dashed line

$$|Y_{env}(f)| = |X_{env}(f)||G(jf)|$$

g. Draw in by hand the spectral lines for $y(t)$
h. Compare the amplitudes of the spectral lines of $x(t)$ and $y(t)$
i. Make use of your result in part (h) to explain why $x(t)$ and $y(t)$ look so much alike
j. Use Matlab to obtain the magnitudes in dBV of the first ten spectral lines of the one-sided spectral plot of $y(t)$

5. Again suppose $x(t)$ is a 1/2 duty cycle pulse train of amplitude $h = 2$ and period $T = 1$ msec but this time suppose that the RC time constant τ is related the pulse width a by $a = 2\tau$
   a. Find C so that $a = 2\tau$
   b. Sketch what you expect the circuit's response to the pulse train looks like
   c. Use time domain arguments involving τ to explain your graph in part (b)
   d. Use Matlab to plot the magnitude of the envelope $Y_{env}(f)$ as follows with a dashed line

$$|Y_{env}(f)| = |X_{env}(f)||G(jf)|$$

   e. Draw in by hand the spectral lines for $y(t)$
   f. Compare the amplitudes of the spectral lines of $x(t)$ and $y(t)$
   g. Make use of your result in part (f) to explain why $x(t)$ and $y(t)$ look different
   h. Use Matlab to obtain the magnitudes in dBV of the first ten spectral lines of the one-sided spectral plot of $y(t)$

6. Now suppose the RC circuit is the same as in Problem (4) but now the pulse train is 1/4 duty cycle with $h = 2$ and period $T = 1$ msec
   a. Sketch what you expect the circuit's response to the pulse train looks like
   b. Make use of time domain arguments involving τ to explain your graph in part (a)
   c. Use Matlab to plot the magnitude of the envelope $Y_{env}(f)$ as follows with a dashed line

$$|Y_{env}(f)| = |X_{env}(f)||G(jf)|$$

   d. Draw in by hand the spectral lines for $y(t)$
   e. Compare the amplitudes of the spectral lines of $x(t)$ and $y(t)$
   f. Make use of your result in part (e) to explain why $x(t)$ and $y(t)$ look different
   g. Use Matlab to obtain the magnitudes in dBV of the first ten spectral lines of the one-sided spectral plot of $y(t)$

LAB

1. The objective these measurements is to see how a first order RC filter with $a = 5\tau$ affects a 1/2 duty cycle pulse train with amplitude $h = 2$ and period $T = 1$ msec
   a. Make a screen capture of the input pulse train $x(t)$
   b. Make a screen capture of the circuit's pulse train response $y(t)$
   c. Compare $x(t)$ and $y(t)$
   d. Obtain and make a screen capture of the one-sided spectral plots of $x(t)$ and $y(t)$
   e. Use Excel to record your measured magnitudes $c_0 - c_{10}$ of the first ten harmonics in dBV

   of $x(t)$ and $y(t)$
   f. Describe and explain the differences between the spectral plots of $x(t)$ and $y(t)$

2. The objective of these measurements is to see how a first order RC filter with $a = 2\tau$ affects a 1/2 duty cycle pulse train with amplitude $h = 2$ and period $T = 1$ msec
   a. Make a screen capture of the input pulse train $x(t)$
   b. Make a screen capture of the circuit's pulse train response $y(t)$
   c. Compare $x(t)$ and $y(t)$
   d. Obtain and make a screen capture of the one-sided spectral plot of $y(t)$
   e. Use Excel to record your measured magnitudes $c_0 - c_{10}$ of the first ten harmonics in dBV of $y(t)$
   f. Describe and explain the differences between the spectral plots of $x(t)$ and $y(t)$

3. The objective of these measurements is to see how the RC circuit of Problem (1) responds to a 1/4 duty cycle pulse train with $h = 2$ and period $T = 1$msec
   a. Make a screen capture of the input pulse train $x(t)$
   b. Make a screen capture of the circuit's pulse train response $y(t)$
   c. Compare $x(t)$ and $y(t)$
   d. Obtain and make a screen capture of the one-sided spectral plot of $y(t)$
   e. Use Excel to record your measured magnitudes $c_0 - c_{10}$ of the first ten harmonics in dBV of $y(t)$
   f. Describe and explain the differences between the spectral plots of $x(t)$ and $y(t)$

POSTLAB

1. Compare your calculated and measured results for the magnitudes of the spectral lines of $y(t)$ at the output of the RC filter in Lab Problem (1) when the input is a 1/2 duty cycle pulse train with $a = 5\tau$, $h = 2$ and period $T = 1$ msec

2. Compare your calculated and measured results for the magnitudes of the spectral lines of $y(t)$ at the output of the RC filter in Lab Problem (2) when the input is a 1/2 duty cycle pulse train with $a = 2\tau$, $h = 2$ and period $T = 1$ msec

3. In Lab Problem (2) we increased τ from $\tau = a/5$ to $\tau = a/2$
   a. Describe how this affected the output of the filter
   b. Use time domain arguments to explain what happened
   c. Use frequency domain arguments to explain what happened - make use of the spectral plots of $x(t)$ and $y(t)$

4. Compare your calculated and measured results for the magnitudes of the spectral lines of $y(t)$ at the output of the RC filter in Lab Problem (3) when the input is a 1/4 duty cycle pulse train with $h = 2$ and period $T = 1$ msec

5. In Lab Problem (3) we decreased the duty cycle of the pulse train from 1/2 to 1/4
   a. Describe how this affected the output of the filter
   b. Use time domain arguments to explain what happened
   c. Use frequency domain arguments to explain what happened - make use of the spectral plots of $x(t)$ and $y(t)$