ECE 257 - LESSON 6 - INTRODUCTION TO MATRIX CALCULATIONS - PART I

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IN CLASS

We define matrix operations like addition, subtraction and multiplication so that we can manipulate and solve systems of linear equations that are in matrix form like the following

$$\begin{pmatrix} 2 & 7 \\ -3 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

"just like" we solve equations of one variable like $2x = 7$.

MATRIX ADDITION AND SUBTRACTION

We *define* matrix addition so that

$$Ax + Bx = (A + B)x$$

is true for systems of equations in matrix form just like it is for equations of one variable. For 2x2 matrices, for example, we define

$$A + B = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \equiv \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + a_{21} & a_{22} + b_{22} \end{pmatrix}$$

Generalizing on this example we define the sum of two matrices with the *same dimensions* to be the sum of the corresponding elements. In the same way we define the difference between two matrices $A - B$ of the same dimensions to be the difference of the corresponding elements.

1. Matrix addition and subtraction

   A = [2 3; 1 4]
   B = [1 2; 2 3]
   C1 = A + B
   C2 = A - B

   a. What happens when we add and subtract matrices

2. When matrix addition and subtraction are not defined

   A = [1 2]
   B = [2 1; 3 -2]
   C = A + B

   a. What has to be satisfied before we can add or subtract two matrices

3. Matrices and numbers

   A = [2 3; 1 4]
   B = [1 2; 2 3]
   C1 = 2*A - B
   C2 = A + B + 2

   a. What happens when we multiply a scalar times a matrix
   b. What happens when we add a scalar to a matrix

MATRIX MULTIPLICATION

Analogously to the case of matrix addition and subtraction we define matrix multiplication so that

$$y = Aw \quad \text{and} \quad w = Bx \quad \text{implies that} \quad y = ABx$$

for systems of equations in matrix form just like it does for equations of one variable. For example suppose

$$\begin{aligned} y_1 &= a_{11}w_1 + a_{12}w_2 \\ y_2 &= a_{21}w_1 + a_{22}w_2 \end{aligned} \qquad \text{and} \qquad \begin{aligned} w_1 &= b_{11}x_1 + b_{12}x_2 \\ w_2 &= b_{21}x_1 + b_{22}x_2 \end{aligned}$$

Then by direct substitution we have

$$\begin{aligned} y_1 &= \left(a_{11}b_{11} + a_{12}b_{21}\right)x_1 + \left(a_{11}b_{12} + a_{12}b_{22}\right)x_2 \\ y_2 &= \left(a_{21}b_{11} + a_{22}b_{21}\right)x_1 + \left(a_{21}b_{12} + a_{22}b_{22}\right)x_2 \end{aligned}$$

Which in matrrix form is equal to

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

We therefore define the matrix product AB to be the coefficient of the vector x in our equation $y = ABx$ as follows

$$C = AB = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \equiv \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix}$$

This looks complicated at first but in fact it's not with

$c_{11} =$ sum of the products when the elements in the first row of A are multiplied by the first column of B and then added together

$c_{12} =$ sum of the products when the elements in the first row of A are multiplied by the second column of B and then added together

and so on. Note that the product AB can be calculated *only if* A has the same number of columns as B has rows.

4. Multiplying square matrices

```
A = [2  3;  1  4]
B = [1  2;  2  3]
C1 = A*B
C2 = B*A
```

   a. What happens when we multiply two matrices
   b. What has to be satisfied before we can multiply two matrices

5. The identity matrix I

```
A = [2  3;  1  4]
I = eye (2)
C1 = A*I
C2 = I*A
```

   a. What happens when we multiply a matrix A by the identity matrix

6. Some simple matrix factoring

```
A = [2  3;  1  4]
B = [1  2;  2  3]
I = eye (2)
C1 = A*B + A
C2 = A*B + A*I
C3 = A*(B + I)
```

7. The distributive law

```
A = [2  3;  1  4]
B = [1  2;  2  3]
C = [2  1;  3  1]
D1 = A*B + A*C
D2 = A*(B + C)
```

   a. How is the distributive law for matrices like the distributive law in regular algebra

8. More matrix multiplication

```
A = [1  2]
B = [3  4]'
C1 = A*B
C2 = B*A
C3 =  A'*B'
C4 =  B'*A'
C5 = A*B'
```

   a. How does the dimension of A have to be related to the dimension of B for the product A*B to be defined
   b. How is the condition for A*B to exist different from the condition for A+B to exist

INVERSE MATRICES AND MATRIX DIVISION

9. The inverse matrix

```
A = [2  3;  1  4]
B1 = inv (A)
B2 = inv (A)*A
B3 = A*inv (A)
```

   a. What does inv(A) do. How can you tell
   b. Can we take the inverse of a nonsquare matrix

10. Solving $Ax = b$ by calculating $x = A^{-1}b$

```
A = [2  3;  1  4]
b = [2  3]'
x = inv (A)*b
A*x
```

   a. How did we use the inverse of A to calculate x

11. Parallel lines

```
A = [1  2;  3  6]
b = [2  3]'
x = inv (A)*b
```

a. What happened when we tried to calculate the intersection of two parallel lines

12. Faster solving of $Ax = b$

```
A = [2  3;  1  4]
b = [2  3]'
x = A\b
```

a. Why is $x = A \backslash b$ faster than $x = A^{-1}b$