ECE 257 - LESSON 23
COMPLEX NUMBERS - PART II

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IN CLASS

COMPLEX EXPONENTIALS

As we saw in the last Lesson it's pretty straightforward to define what we mean by the addition, multiplication and so on of complex numbers. A more challenging problem is how to define complex functions like the following complex exponential

$$z = 2^{3+j4} = x + jy$$

Now, Euler's genius was not only that he figured out a way to **define** complex exponentials $z = c^{jb}$

(1) So they would have the same really nice properties that real exponentials do like

$$x^a x^b = x^{a+b} \text{ and } \frac{x^a}{x^b} = x^{a-b}$$

(2) But also satisfy **Euler's Relation** as follows for the special case of $c = e$

$$e^{jb} = \cos(b) + j\sin(b)$$

Euler accomplished this fete by taking the Taylor's Series Expansion for real exponentials c^x as follows

$$c^x = 1 + (\ln c)\, x + \frac{(\ln c)^2}{2!}\, x^2 + \frac{(\ln c)^3}{3!}\, x^3 + \cdots$$

and making the substitution $x = jb$ to obtain his definition for complex exponentials as follows

$$c^{jb} \equiv 1 + (\ln c)(jb) + \frac{(\ln c)^2}{2!}(jb)^2 + \frac{(\ln c)^3}{3!}(jb)^3 + \cdots$$

$$= 1 + j(\ln c)b - \frac{(\ln c)^2}{2!}\, b^2 - j\,\frac{(\ln c)^3}{3!}\, b^3 + \cdots$$

With $c = e$ we then have

$$e^{jb} = 1 + j(\ln e)b - \frac{(\ln e)^2}{2!}\, b^2 - j\,\frac{(\ln e)^3}{3!}\, b^3 + \cdots$$

$$e^{jb} = \left(1 - \frac{b^2}{2!} + \frac{b^4}{4!} - \cdots\right) + j\left(b - \frac{b^3}{3!} + \cdots\right)$$

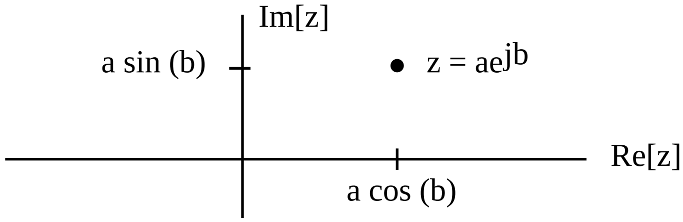
which gives us Euler's Relation

$$e^{jb} = \cos(b) + j\sin(b)$$

Therefore

$$z = ae^{jb} = a\cos(b) + ja\sin(b)$$

is the complex number with rectangular coordinates $x = a\cos(b)$ and $y = a\sin(b)$ located in the complex plane as follows with polar coordinates $r = a$ and $\theta = b$



1. Some complex exponential calculations

```
>>  z = 2*exp(j*3)
>>  x = 2*cos (3)
>>  y = 2*sin (3)
>>  r = abs (2*exp(j*3))
>>  theta = angle (2*exp(j*3))
```

   a. What are the rectangular coordinates of the complex exponential ae^{jb}
   b. What are the polar coordinates of the complex exponential ae^{jb}

2. More complex exponential calculations

```
T = 1e-3;
f = 1/T;
t = linspace (0,  3*T,  500);
x = real (5*exp(j*2*pi*f*t));
plot (t, x)
```

   a. Describe the result of this code