ECE 257 - LESSON 22
COMPLEX NUMBERS - PART I

SPRING 2007 **A.P. FELZER**

IN CLASS

INTRODUCTION TO COMPLEX NUMBERS

Real numbers are great. They're great because we can use them to count and keep track of everything from the number of students in a class to the amount of money in a bank account. They're great because they have particularly nice properties like the following

$$a + b = b + a \qquad ab = ba \qquad a(b + c) = ab + ac$$

And they're great because they're the solutions to equations like the following polynomial

$$x^2 + x - 2 = 0$$

But real numbers cannot solve all equations. In particular there are no real solutions to the equation

$$x^2 + 1 = 0 \qquad x^2 = -1 \qquad x = \pm\sqrt{-1}$$

since $x = \sqrt{-1}$ is not a real number.

The way we get around this impasse is to simply allow imaginary numbers $j = \sqrt{-1}$ and their more general cousins of the form $z = a + jb$ into the *club of numbers*. Now these complex numbers can't be used to measure distances between objects or to keep track of the amount of money we have in the bank but they're still bona fide numbers with all the properties of real numbers like those above as follows

$$a + b = b + a \qquad ab = ba \qquad a(b + c) = ab + ac$$

And they're very useful in real world applications especially in simplifying calculations in electrical engineering.

SOME COMPLEX CALCULATIONS

1. Some basic calculations

>> $z1 = sqrt(-1)$
>> $z2 = 2j$
>> $z3 = j2$
>> $z4 = j \quad 2$
>> $z5 = sqrt(1 + j \quad 2)$
>> $z6 = \text{conj}(2 + j \quad 3)$
>> $z7 = j \quad \cos(2)$

a. When do we need a multiplication sign for j
b. What does the function conj do

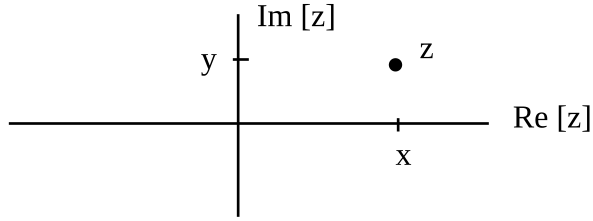
2. Some more calculations

>> $z1 = 2 + j \quad 3$
>> $z2 = 3 - j \quad 5$

```
>>   za = z1   z2
>>   zb = z1/z2
```

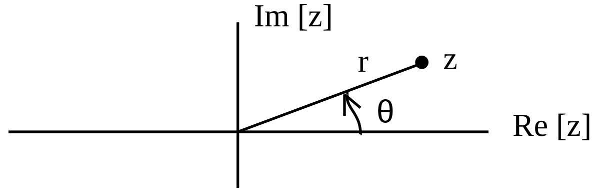
THE COMPLEX PLANE

It took mathematicians a long time to get used to complex numbers $z = x + jy$ but they're acceptance grew once people realized they could be plotted in a **complex plane** as follows



where x and y are referred to as the **rectangular coordinates** of the complex number z

Alternatively we can specify the location of a complex number in the complex plane with its **polar coordinates** r and θ as follows



You should **memorize** the fact that we refer to

$r = |z|$ as the **magnitude** of z and

$\theta = \quad z$ as the **phase** or **phase angle** of z

3. Converting between rectangular and polar coordinates

```
>>  z = 3 + j   2
>>  r = abs(z)
>>  theta = angle(z)
>>  zx = real(z)
>>  zx = r   cos(theta)
>>  zy = imag(z)
>>  zy = r   sin(theta)
```

   a. How are the polar and rectangular coordinates of a complex number related

4. Distance between complex numbers in the complex plane

```
>>  z1 = 2 + j*3;
>>  z2 = 5 + j*7;
>>  dist = sqrt ((real(z1) - real(z2))^2 + (imag(z1) - imag(z2))^2)
>>  dist = abs (z1 - z2)
```

   a. How can we calculate the distance between two complex numbers

PLOTTING THE MAGNITUDES OF COMPLEX RATIONAL POLYNOMIALS

5. Plotting of $|G| = \left|\dfrac{2000}{j2\pi f + 1000}\right|$ as a function of f from $f = 10$ Hz to $f = 10^5$ Hz

```
p1 = -1000;
f = logspace (1, 5, 200);
G = 2000./(j*2*pi*f - p1);
mag_G = abs (G);
semilogx (f, mag_G)
xlabel ('f on a log scale');
ylabel ('Magnitude of G');
title ('Magnitude of G as a function of frequency f');
```