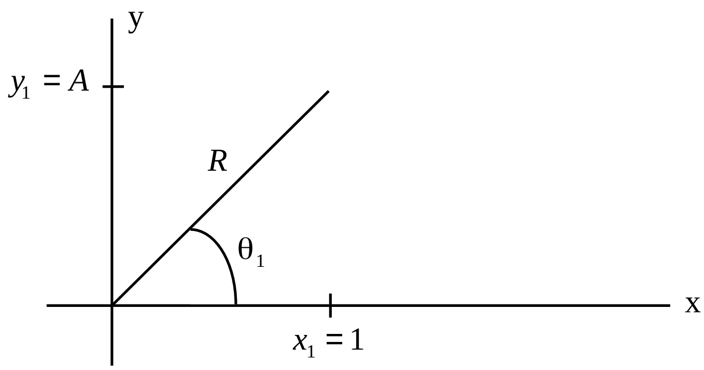
ECE 257 - LESSON 19
INTRODUCTION TO ITERATION - PART II

SPRING 2007 **A.P. FELZER**

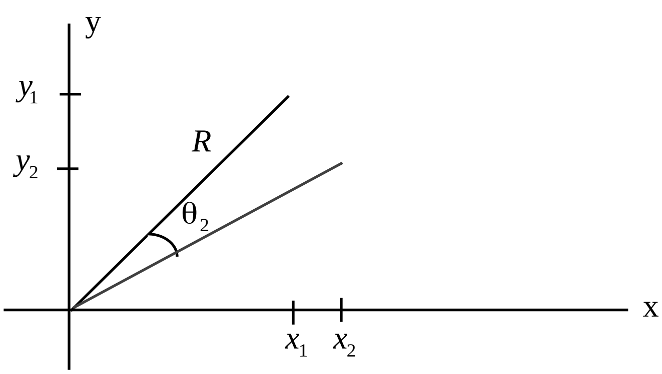
IN CLASS

The following algorithm comes from an article in a 1977 Hewlett-Packard Journal by William Egbert that describes how HP calculators iteratively calculate inverse tangets $\theta_1 = \tan^{-1} A$ as shown in the following drawing



The idea of the algorithm is as follows:

(1) Repeatedly rotate the line clockwise by an angle θ_2 as follows



until y_2 becomes negative

(2) Then back up one rotation so y_2 is positive again and continue the clockwise rotation but now with a smaller angle θ_3

(3) And so on until we have the angle θ_1 equal to a sum like the following

$$\theta_1 = \theta_2 + 2\theta_3 + \theta_4$$

with the last theta less than or equal to the desired resolution

To carry out this algorithm we need equations for x_2 and y_2 in terms of x_1 and y_1. We get these equations from the fact that

$$x_2 = R\cos(\theta_1 - \theta_2) = R\cos(\theta_1)\cos(\theta_2) + R\sin(\theta_1)\sin(\theta_2)$$

and so

$$x_2 = \cos(\theta_2)\left(R\cos(\theta_1) + \frac{R\sin(\theta_1)\sin(\theta_2)}{\cos(\theta_2)}\right) = \cos(\theta_2)\left(x_1 + y_1\tan(\theta_2)\right)$$

And analogously

$$y_2 = \cos(\theta_2)\left(y_1 - x_1\tan(\theta_2)\right)$$

Now since dividing both x_2 and y_2 by the same amount doesn't affect the angle we implement the algorithm with the modified values x_2' and y_2' as follows

$$x_2' = \frac{x_2}{\cos(\theta_2)} = x_1 + y_1\tan(\theta_2) \qquad \text{and} \qquad y_2' = \frac{y_2}{\cos(\theta_2)} = y_1 - x_1\tan(\theta_2)$$

Note that the angles $\theta_2,\ \theta_3, \ldots$ used in the algorithm are chosen by HP to have nice simple tangents as follows

$$\tan(\theta_2) = 1, \quad \tan(\theta_3) = 0.1, \quad \tan(\theta_4) = 0.01, \quad \ldots$$

in order to make the calculations as easy as possible.

Be sure to note that once the values of the tangents are chosen to be $\tan(\theta_2) = 1$, $\tan(\theta_3) = 0.1$, $\tan(\theta_4) = 0.01, \ldots$ and then the corresponding angles $\theta_2 = \tan^{-1}(1)$, $\theta_3 = \tan^{-1}(0.1), \ldots$ are calculated and stored away in a vector then *no tangents or inverse tangents* are calculated as the program iterates