ECE 257 - LESSON 18
INTRODUCTION TO ITERATION - PART I

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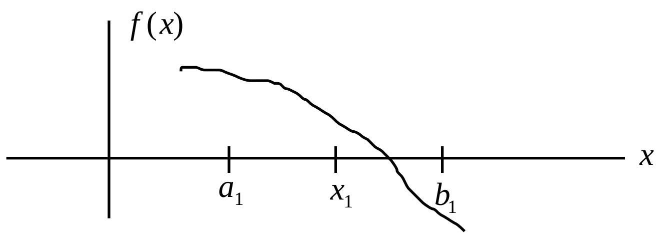
IN CLASS

Iterative algorithms are algorithms that step by step get us closer and closer to a problem's solution very much like the questions in the game "twenty questions." Iterative algorithms are usually much more efficient than the brute force methods we've been using to find things like the zeroes and the maximums of functions.

1. The goal of this problem is to show how iteration can be used to find where a function $f(x)$ like the following crosses the x-axis between a_1 and b_1 as follows



The goal of each iteration is to narrow down the interval where $f(x)$ crosses the x-axis as follows:

Iteration 1: First calculate $f(x)$ $x = a_1$ and then at at $x_1 = \frac{1}{2}(a_1 + b_1)$ half way between a_1 and b_1. Then there are two possibilities:

(1) If $f(a_1)$ and $f(x_1)$ are both positive or both negative then we can conclude that $f(x) = 0$ is somewheres in the smaller interval $x_1 \le x \le b_1$ bounded by

$$a_2 = x_1 \quad \text{and} \quad b_2 = b_1$$

(2) But if $f(a_1)$ and $f(x_1)$ have different signs then . . .

Iteration 2: Repeat the steps of Iteration (1) but this time for the smaller interval with end points a_2 and b_2 and midpoint $x_2 = \frac{1}{2}(a_2 + b_2)$

Iterations 3, . . . : Keep iterating until the interval is narrowed down to the desired resolution. The middle of this interval is then the solution to $f(x) = 0$ to the desired resolution

Use this algorithm to iteratively find where $f(x) = x^2 - x - 3.75 = 0$ to within a resolution of 10^{-2} in the interval $0 \le x \le 4$. Test your result by verifying that $f(x)$ is closer to 0 in the middle of the final interval than it is at the edges. Put your results in a Table.