ECE 109 - KIRCHHOFF'S LAWS - INVESTIGATION 8
KIRCHHOFF'S VOLTAGE LAW

SUMMER 2007 **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

From the last Investigation on KCL we know the following:

Big Observation: Equivalent positive charges enter nodes at exactly the same rate as they leave

Big Consequence: The algebraic sum of the currents at a node is zero. We call this Kirchhoff's Current Law

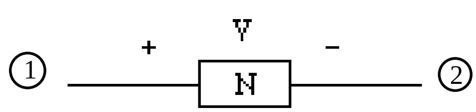
The objective of this Investigation is to come up with equations for the algebraic sums of voltages around closed loops analogous to Kirchhoff's Current Law at nodes. We will be making use of the fact that the voltage drop V across a circuit element N tells us the amount of energy being transferred between N and the equivalent positive charges flowing through it in units of

volts = joules/coulomb

We will also be using our result that equivalent positive charges going from a higher to a lower potential are transferring energy to N while those going from a lower to a higher potential are receiving energy from N.

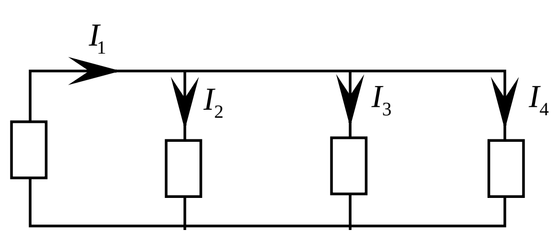
Be sure to take a look at the **Computer Demos** on Kirchhoff's Voltage Law.

1. We begin with some review problems. Given the following circuit element N with V = –5 volts



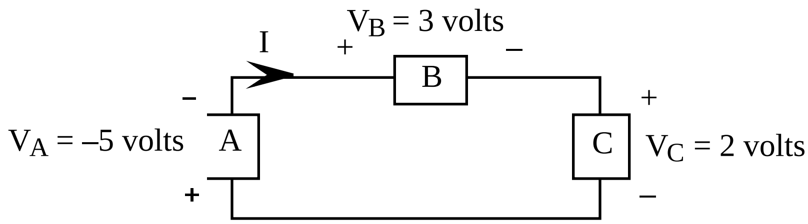
a. At which node will the equivalent positive charges have more energy
b. Will epc flowing from node 1 to node 2 be receiving energy from N or transferring energy to it
c. Will epc flowing from node 2 to node 1 be receiving energy from N or transferring energy to it

2. Find I_1 as a function of I_2, I_3 and I_4 in the following circuit. **Memorize** your result



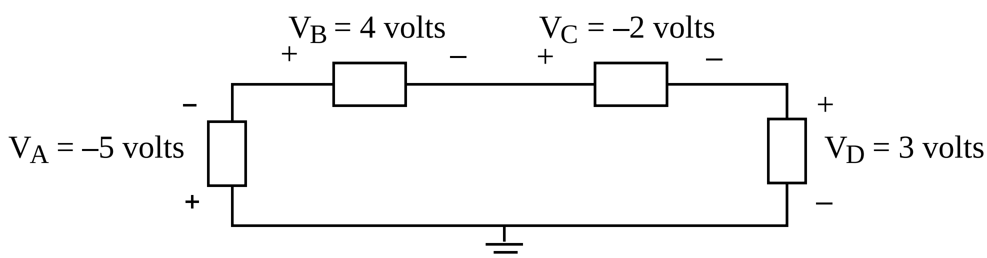
3. Now from the last Investigation we know that Kirchhoff's Current Law at nodes is based on the fact that charges always flow out of nodes at the same rate they flow in. The corresponding *fundamental result* for voltages is based on what happens to the energy the charges get from

the sources as they flow around closed loops. As always we must start with results from the lab. Let's suppose, in particular, that we measure the following voltages



a. Make use of the measured voltages to determine how much energy is being received by Q = 2 coulombs of equivalent positive charge flowing clockwise around the circuit
b. Then make use of the measured voltages to determine how much energy is being delivered by Q = 2 coulombs of equivalent positive charge flowing clockwise around the circuit
c. What is the net energy transfer between the circuit elements and the equivalent positive charges flowing through them - how is the energy of equivalent positive charge starting from the reference be related to its energy after it goes around the circuit

4. Generalizing on the result of Problem (3) it can be shown that equivalent positive charges flowing around a circuit always return with the same energy. The objective of this and the next several problems is to explore the consequences of this result. We begin with the special case of a circuit with all voltage reference directions **aligned** from plus to minus as we go clockwise around the circuit as follows



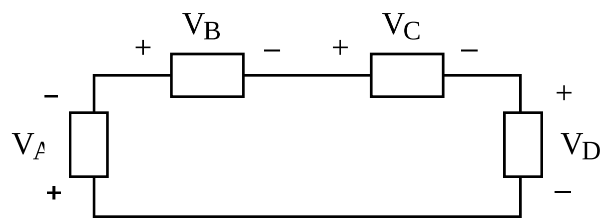
a. Find the sum of the positive voltages.
b. Find the sum of the negative voltages.
c. Now explain in words why

$$\sum \text{(Positive Voltages)} + \sum \text{(Negative Voltages)} = 0$$

for this circuit with *aligned* voltage reference directions. Hint - first explain why the two sums are equal in magnitude and then why they're opposite in sign.

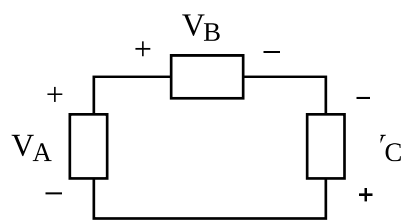
5. Generalizing on the results of Problem (4) we can say that the summation of all the voltages around a closed loop are zero when the reference directions are aligned because the equivalent positive charges flowing around the loop are receiving exactly as much energy from the sources as they're delivering to the resistors. **Memorize** this result.

Suppose in particular that equivalent positive charge flowing clockwise around the following circuit receive a total of 5 joules/coulomb. Assuming all the reference signs are *aligned* as follows

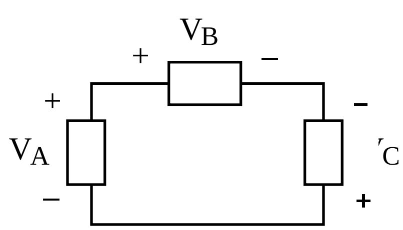


a. How many joules/coulomb did the equivalent positive charges return to the circuit.
b. What is the sum of the positive voltages.
c. What is the sum of the negative voltages.
d. What is the sum of all the voltages.

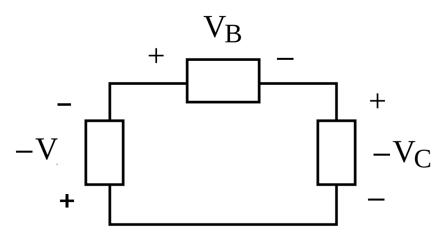
6. The objective of this problem is to find the relationship between the voltage drops in a circuit when they're not all aligned like in the following example



The trick is to make use of the fact that these non-aligned voltage reference directions can be turned around to form an equivalent set of aligned reference directions as follows



is equivalent to



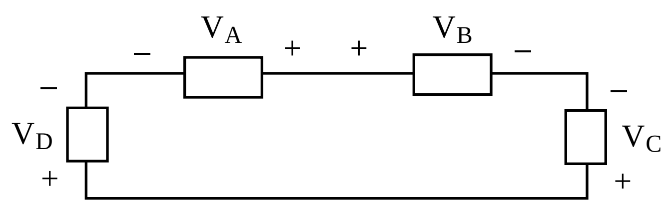
Make use of this result to find the equation relating V_A, V_B and V_C

7. We refer to the equation from Problem (5) as follows

$$-V_A + V_B - V_C = 0$$

as the **algebraic sum of the voltages around the closed loop**.

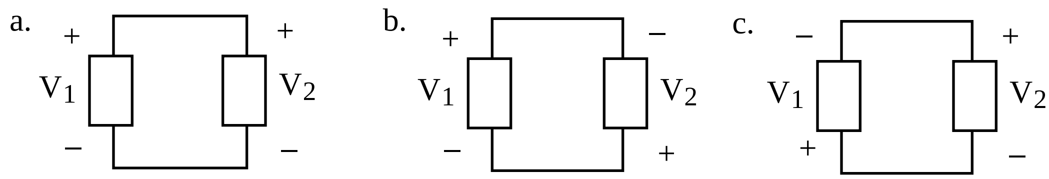
a. Which voltages do we add and which do we subtract when calculating the algebraic sum of the voltages as we go clockwise around a closed loop.
b. Now find the equation for the algebraic sum of the voltages around the following closed loop



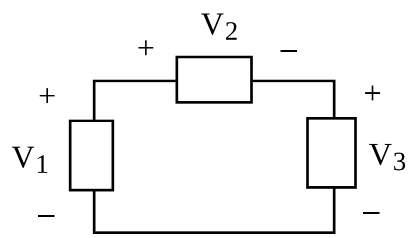
8. Generalizing on the results of Problems (6) and (7) we have that the algebraic sums of voltages around closed loops always add up to zero. We call this result **Kirchhoff's Voltage Law**

(KVL). **Memorize** it forever. Then make use of KVL to find V_B in Problem (7) if V_A = 3 volts, V_C = -2 volts and V_D = 2 volts

9. Find V_2 as a function of V_1 in each of the following circuits. **Memorize** your results

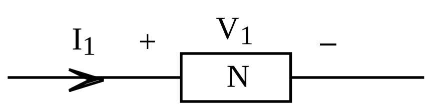


10. Given the following circuit

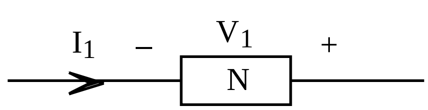


a. Find V_1 in terms of V_2 and V_3. **Memorize** this result
b. Find V_2 in terms of V_1 and V_3. **Memorize** this result

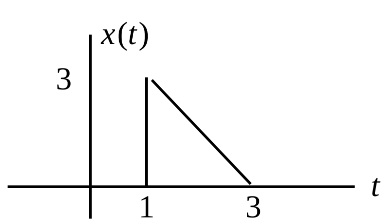
11. Suppose we measure $V_1 = -3$ volts and $I_1 = -5$ ma for the following circuit element



What would we have measured for V_1 and I_1 if their reference directions had been



12. Math Review: Given the following graph for $x(t)$



Sketch each of the following for $t \geq 0$

a. $y_1(t) = 2x(t)$
b. $y_2(t) = x(t) + 2$
c. $y_3(t) = 2x(t) + 2$