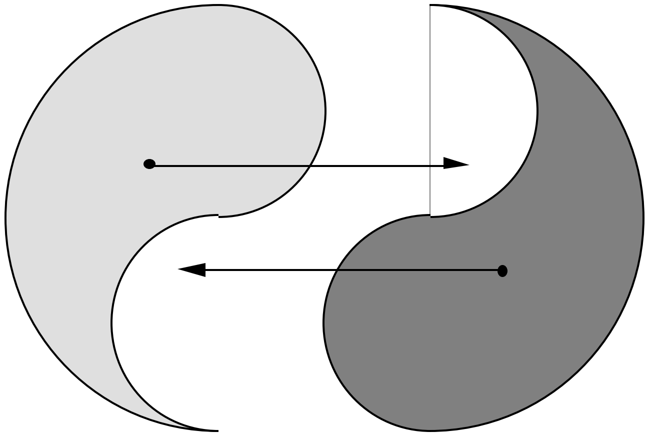
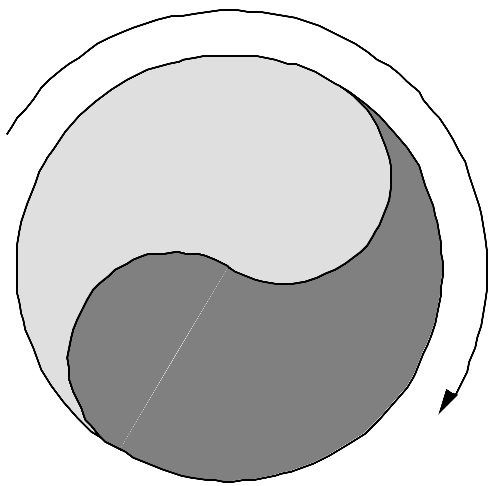
Head on Collision II: Yin-Yang Soup

A. John Mallinckrodt and Harvey S. Leff

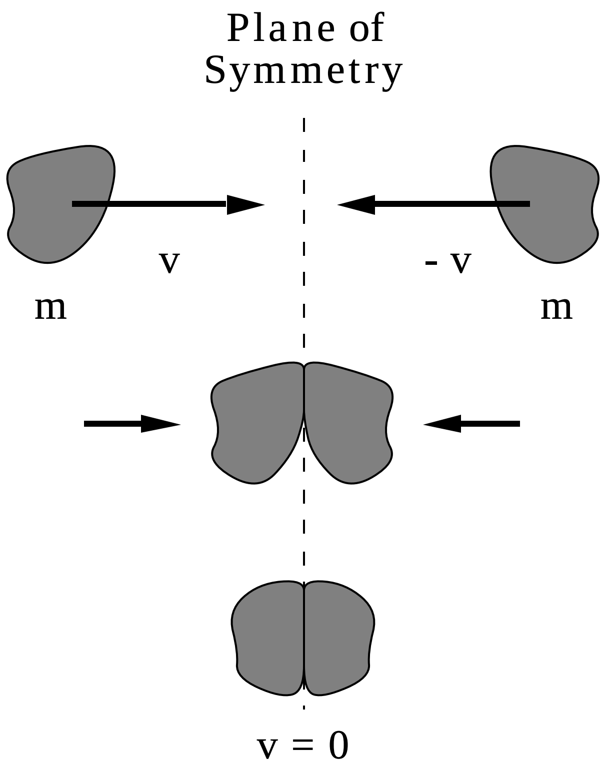
Physics Department, Cal Poly Pomona
3801 W. Temple Avenue, Pomona, CA 91768

Is this a "totally inelastic" collision?





Mirror-symmetric totally inelastic collision in system CM frame



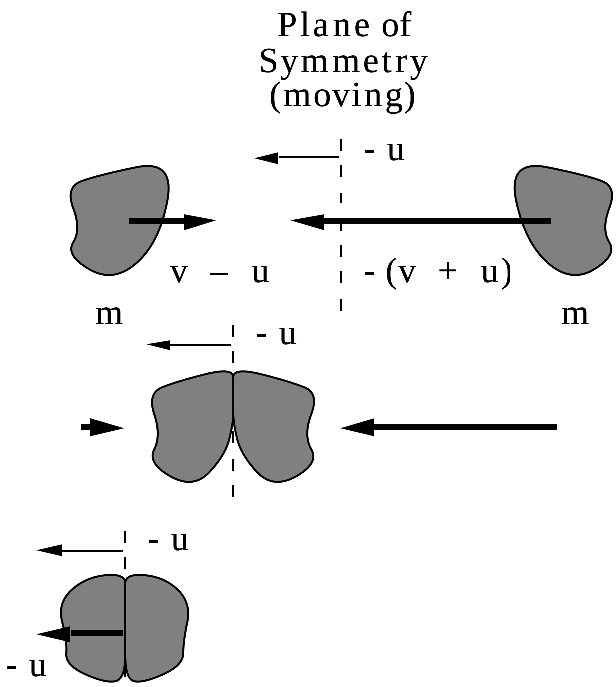
Symmetry	$W_1 = W_2$
Newton's third law + contact force assumption	$W_1 = -W_2$

Taken together these imply that $W_1 = W_2 = 0$;
i.e., *both objects are brought to rest while zero work is done on them.*

Introducing ... Internal Energy

$$W = \Delta K + \Delta U$$

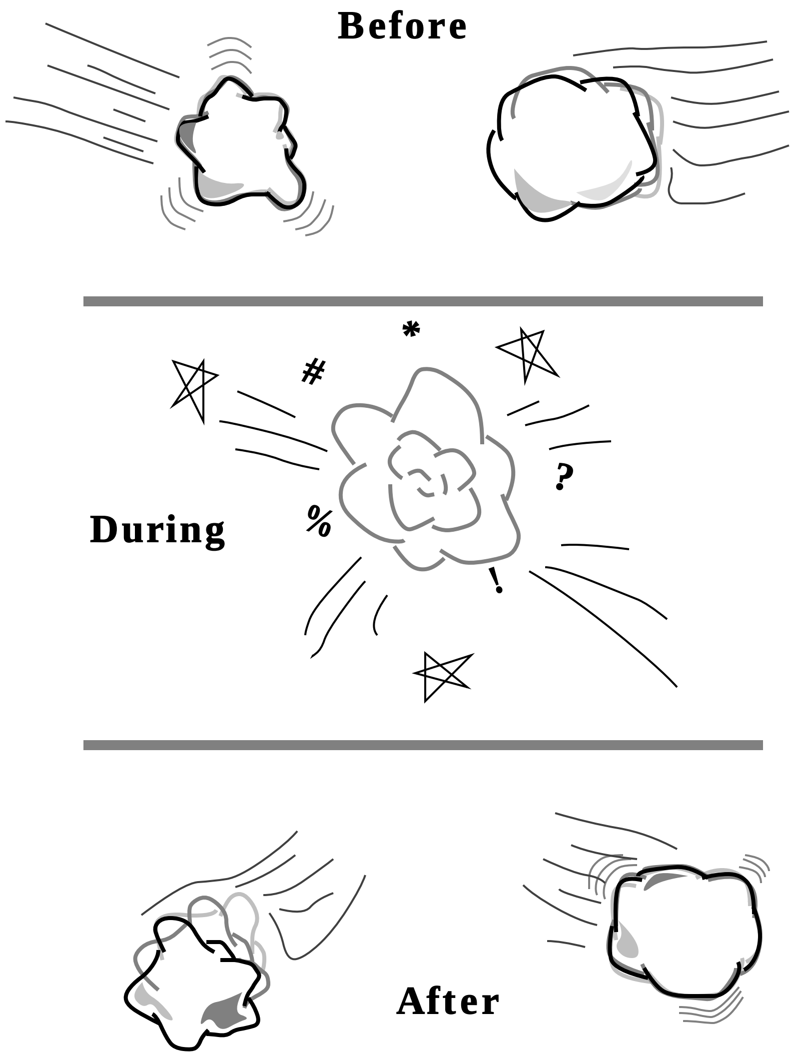
Can show that, though W and ΔK are frame-dependent, ΔU (*whatever* it is) is not. Consider a frame moving at "velocity" u with respect to the system CM:



$W_1 > 0$, $W_2 = -W_1$

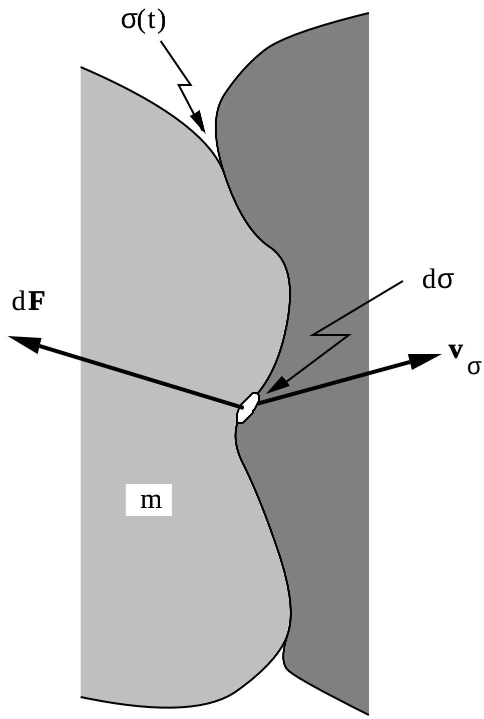
$\Delta U_1 = \Delta U_2 = \frac{1}{2}\, mv^2$ (in *any* frame) *No heat!!!*

A generalized collision



Transformations of ΔK and W in general collisions

Define terms:



$$K \equiv \frac{1}{2} m\mathbf{v}\ \ \mathbf{v}$$

where

m = mass of body

$\mathbf{v}$ = velocity of body CM

$$W \equiv \int_{\substack{\text{duration of}\\ \text{the collision}}} dt \int_{\substack{\text{contact}\\ \text{surface}}} d\mathbf{F}\ \ \mathbf{v}_\sigma$$

where

d$\mathbf{F}$ = force on element of contact surface

$\mathbf{v}_\sigma$ = instantaneous velocity of surface element

Then we can show by carrying out the Galilean transformation

$$\mathbf{v}\ \rightarrow \mathbf{v} - \mathbf{u} \text{ and } \mathbf{v}_\sigma \rightarrow \mathbf{v}_\sigma - \mathbf{u}$$

that

$$\Delta K(\mathbf{u}) = \Delta K(\mathbf{0}) - \mathbf{u}\ \ \Delta\mathbf{p}$$

and

$$W(\mathbf{u}) = W(\mathbf{0}) - \mathbf{u}\ \ \Delta\mathbf{p}$$

Implications

$W_1 = -W_2$ (works on colliding bodies are equal and opposite)

$W = \Delta K + \Delta U$ (generalized work-energy theorem for each body)

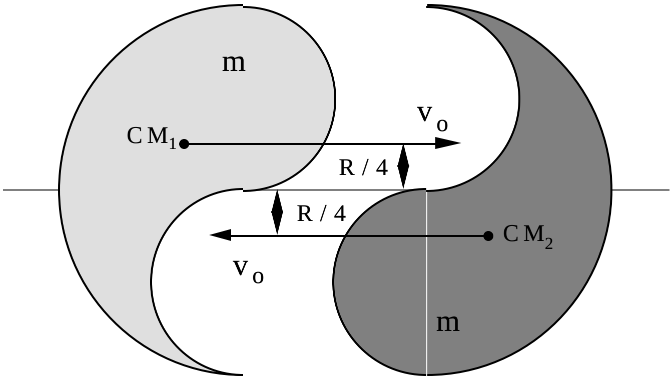
$\Delta K(\mathbf{u}) = \Delta K(\mathbf{0}) - \mathbf{u} \cdot \Delta\mathbf{p}$ (transformation to any frame from system CM frame)

$W(\mathbf{u}) = W(\mathbf{0}) - \mathbf{u} \cdot \Delta\mathbf{p}$ (transformation to any frame from system CM frame)

1. ΔU (= $W - \Delta K$) is *frame-invariant*.
2. If $\Delta\mathbf{p} \neq 0$ then there exists a family of reference frames (with $\mathbf{u} \cdot \Delta\mathbf{p} = W(\mathbf{0})$) in which no work is done on the body (or on its collision partner). In these frames no net energy is transfered from one body to the other. The interaction serves only to facilitate a process in which the energy *distribution* of each body between bulk kinetic and internal forms is altered.

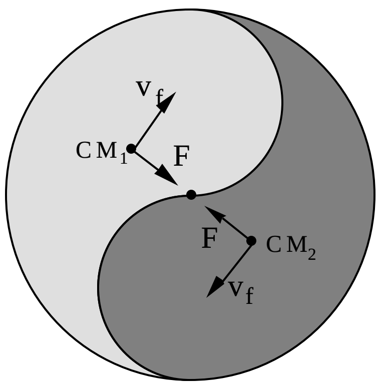
   There also exists a family of reference frames in which $\Delta K_1 = 0$ and another in which $\Delta K_2 = 0$.
3. If $\Delta\mathbf{p} = 0$ then $\Delta K = 0$ and W is frame invariant. $W = \Delta U$ in all reference frames. The net effect of the interaction is only to transfer internal energy from one body to the other.

A second look at the Yin-Yang collision



Each part

$$K_o = \frac{1}{2}\, m\, v_o^2$$



Each part

$$v_f = \frac{\sqrt{1+(4/\pi)^2}}{8}\, v_o$$

$$K_{trans\ f} = \frac{16+\pi^2}{64\pi^2}\, K_o$$

$$K_{rot\ f} = \frac{7\pi^2-16}{64\pi^2}\, K_o$$

$$\Delta U = \frac{7}{8}\, K_o$$

$$F = \frac{\sqrt{16+\pi^2}}{8\pi R}\, K_o$$

1. Each body retains translational kinetic energy.
2. Each body exerts a centripetal force on the other; the surfaces need to be sticky.
3. In the system CM frame no work is done after contact is established since **F** is perpendicular to the direction of motion of each body.
4. In "moving" frames, energy is constantly shuttled back and forth via non-zero-work; the collision never "ends."

Summary

1. The generalized work-energy theorem

$$W \;\; = \Delta K + \Delta U$$
$$+ Q ?$$

(Generalized first law of thermodynamics?)

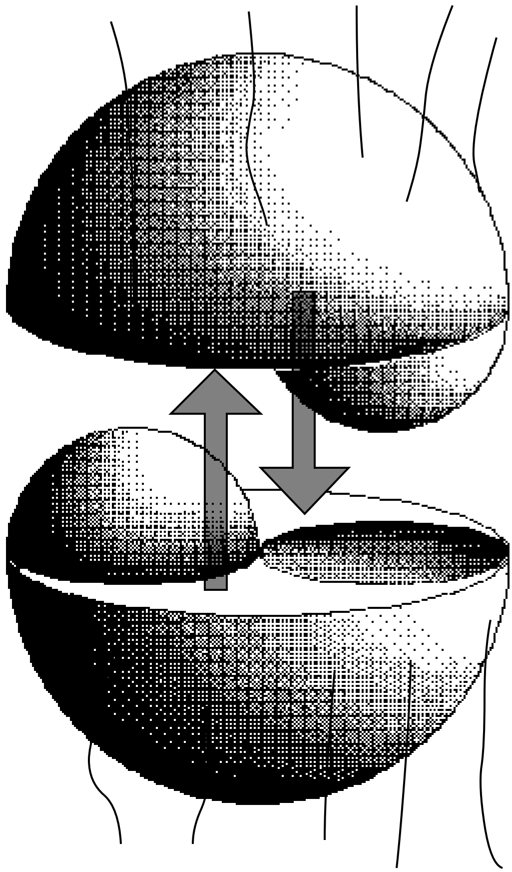
specifies a relationship between the (frame-dependent) change in bulk kinetic energy of a body and the (frame-invariant) change in its internal energy content when it interacts with another body.

2. For most interactions, frames can be found in which $W = 0$ but $\Delta K = -\Delta U \neq 0$. (In the absence of an interaction all *three* terms vanish.) An interaction, then, merely facilitates the transfer of energy between bulk kinetic and internal modes *whether or not work is done.*

3. Because of the general transformation laws for W and ΔK, ΔU is *explicitly* frame-invariant in all interactions.

4. The elasticity or inelasticity of a collision depends on one's viewpoint. All collisions are elastic if you can (and care to) keep track of enough things.

Just for the heck of it?



Work in different reference frames