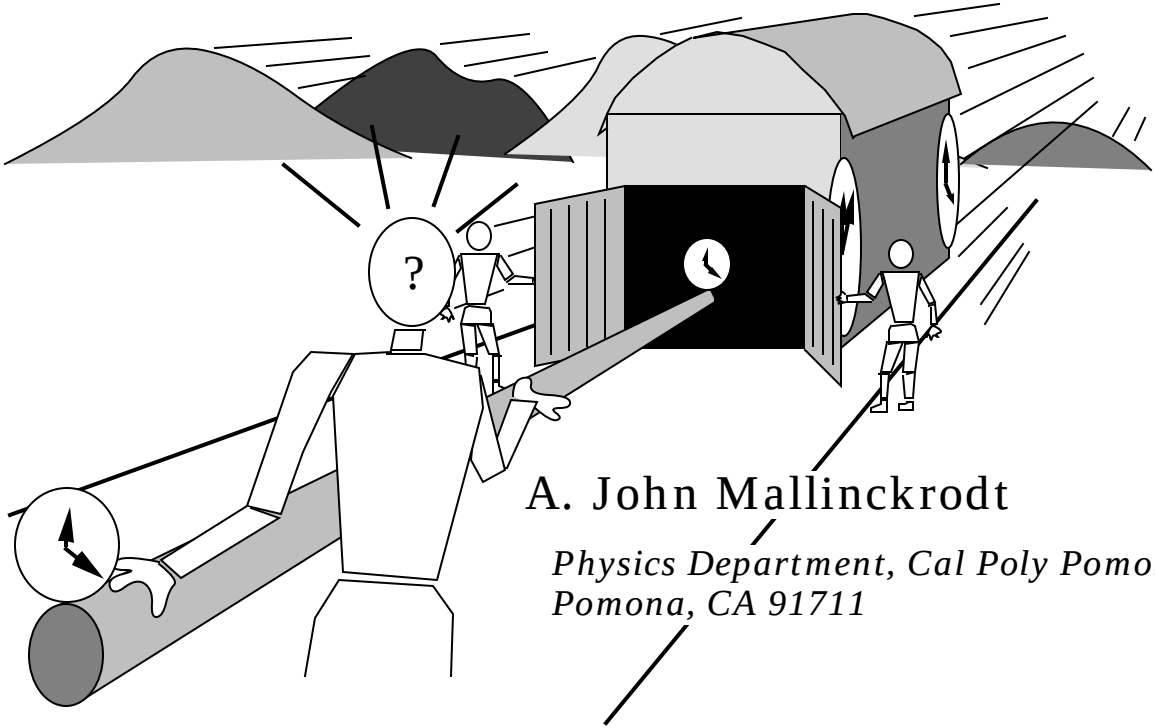
The Pole and the Barn:

A Worksheet on the Consistency of Special Relativity



A. John Mallinckrodt

Physics Department, Cal Poly Pomona
Pomona, CA 91711

Recap of the "paradox"

The pole is longer than the barn.

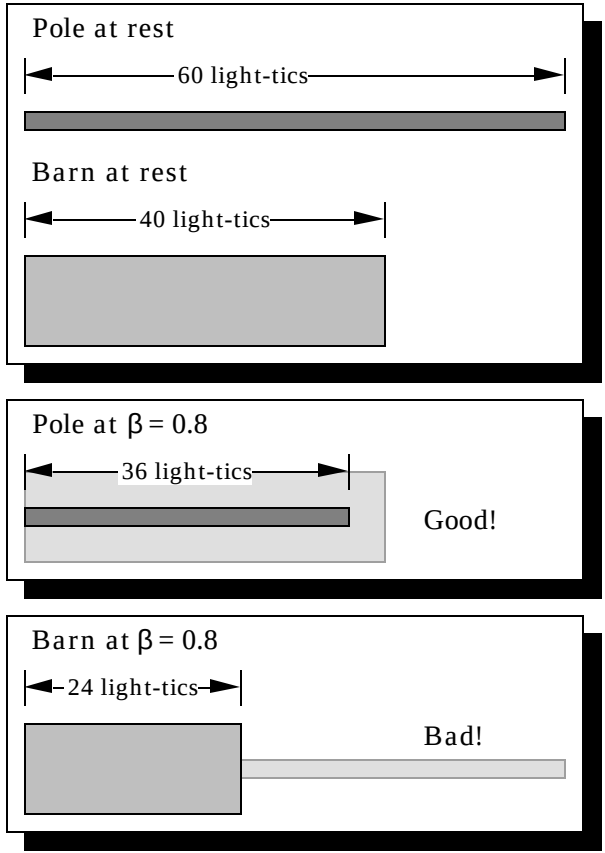
➥ Let L_{barn} = 60 "light-tics"

➥ Let L_{pole} = 40 "light-tics"

Use the Lorentz contraction to fit the moving pole in the barn.

➥ Let $\beta = v/c = 4/5$ ($\gamma = 5/3$)

But what happens as seen from the frame of the pole?



The standard "resolution"

- Trailing edge of the pole *does* get inside the barn as observed by *anybody*. (Results from the relativity of simultaneity.)

- Bonus result: No such thing as a perfectly rigid body.

- Lorentz transformations and spacetime diagrams may be used to obtain and/or illustrate the solution.

An alternate approach

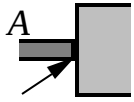
- Consider snapshots of important "instants" of time in both frames.
- Determine the readings of many clocks in each snapshot using only simple relations.
- Perform the analysis *of* both frames *from* both frames.
- Compare the readings of clocks that "participate" in the events in both frames.

Benefits

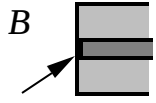
- Emphasizes the crucial relativistic concept of the "event" and distinguishes it from that of the "instant."
- Conveys an appreciation for the internal consistency of relativity.

The Events (as observed in barn frame):

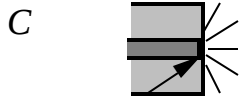
➥ *A* Right end of pole enters barn.



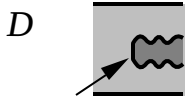
➥ *B* Left end of pole enters barn.



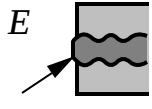
➥ *C* Right end of pole hits back of barn.



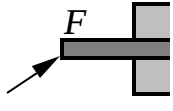
➥ *D* Shock wave hits left end of pole.



➥ *E* Left end of pole emerges from barn.



➥ *F* Left end of pole straightens out.

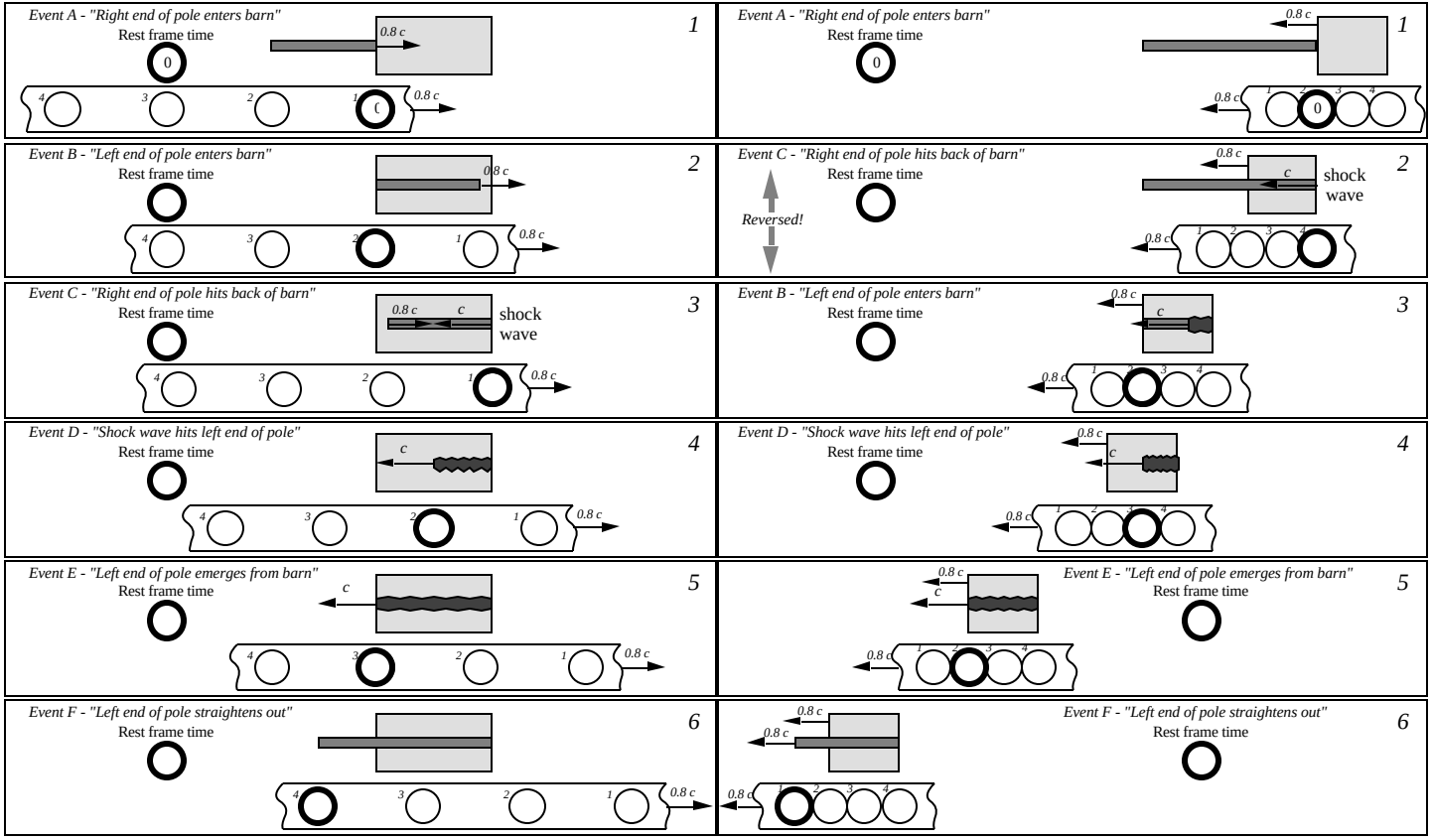


Barn frame

The barn is 40 light-tics long.
The pole has a rest length of 60 light tics and moves to the right at 0.8 c (initially).
Four equally separated clocks move to the right at a constant 0.8 c.
Clocks #1 and #2 travel (initially) with the right and left ends of the pole respectively.

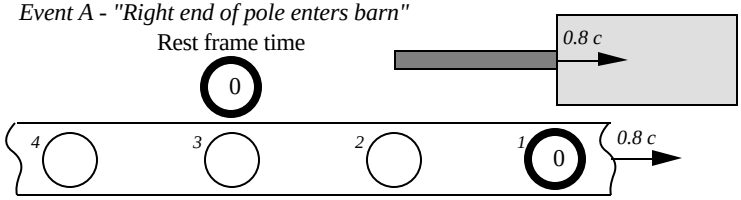
Pole frame

The pole is 60 light-tics long.
The barn has a rest length of 40 light tics and moves to the left at 0.8 c.
Four equally separated clocks move to the left at a constant 0.8 c.
Clocks #2 and #4 travel with the left and right ends of the barn respectively.



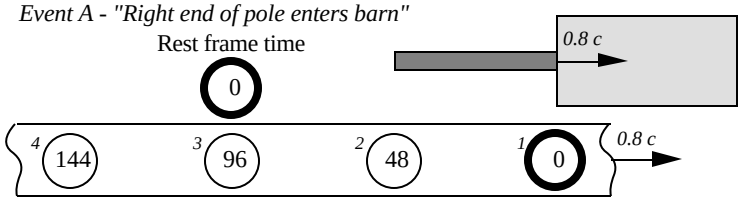
Barn Frame - Event A

- Given: Moving clocks are separated by 60 light-tics (in their own frame) and move with the pole.
- Given: $t_A = 0$ in both frames.

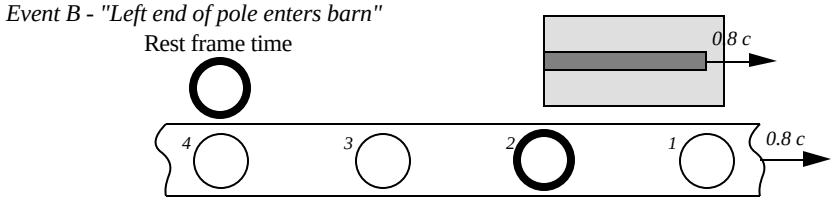


- The pole (and the distance between adjacent clocks) is contracted to $L_o/\gamma = 36$ light-tics
- Adjacent clocks are out of sync by $\beta\Delta L_o/c = 48$ tics and "trailing" clocks read later times.

So...

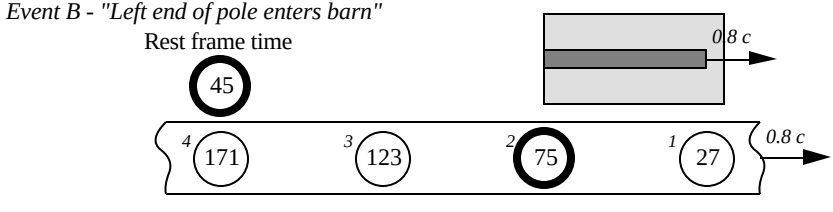


Barn Frame - Event B



- Time elapsed in barn frame
  = distance pole travels/speed of pole
  = 36 light tics/.8 c = 45 tics.
- Time advance of clocks in pole frame
  = 45 tics/γ = 27 tics

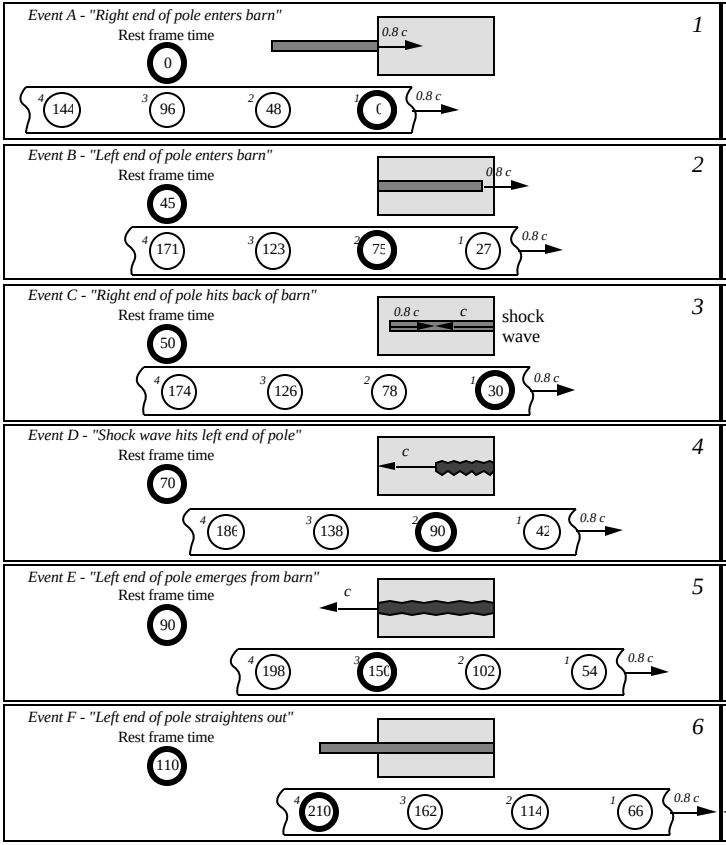
So...



and so on through all six events. Then repeat from the pole frame finding ...

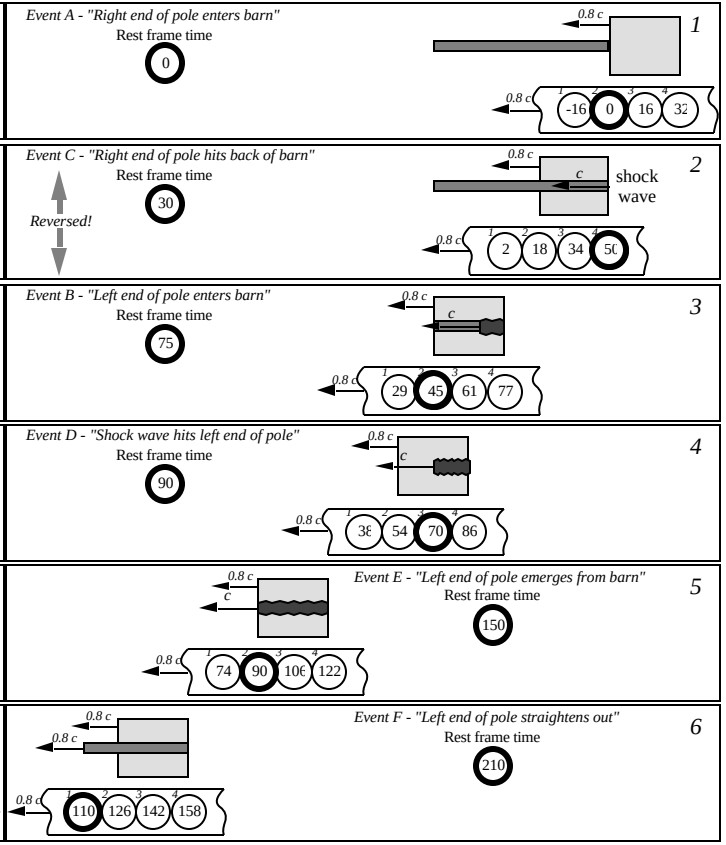
Barn frame

The barn is 40 light-tics long.
The pole has a rest length of 60 light tics and moves to the right at 0.8 c (initially).
Four equally separated clocks move to the right at a constant 0.8 c.
Clocks #1 and #2 travel (initially) with the right and left ends of the pole respectively.



Pole frame

The pole is 60 light-tics long.
The barn has a rest length of 40 light tics and moves to the left at 0.8 c.
Four equally separated clocks move to the left at a constant 0.8 c.
Clocks #2 and #4 travel with the left and right ends of the barn respectively.



Conclusions

- The worksheet provides a concrete, visual approach to a significant problem in relativity.
- Only simple concepts are required for the analysis:
  - » Distance = speed x time.
  - » Moving objects are contracted and moving clocks run slow, both by the relativistic factor γ.
  - » Moving clocks are out of sync by an amount $\beta L_0/c$ with trailing clocks reading later times.
- The results are more convincing and they more directly illustrate the consistency of relativity than approaches based on the Lorentz transformations or spacetime diagrams.
- The exercise is a satisfying capstone experience for students finishing their introduction to special relativity.