Diving in Artificial Gravity: A Memorable Lesson on Fictitious Forces

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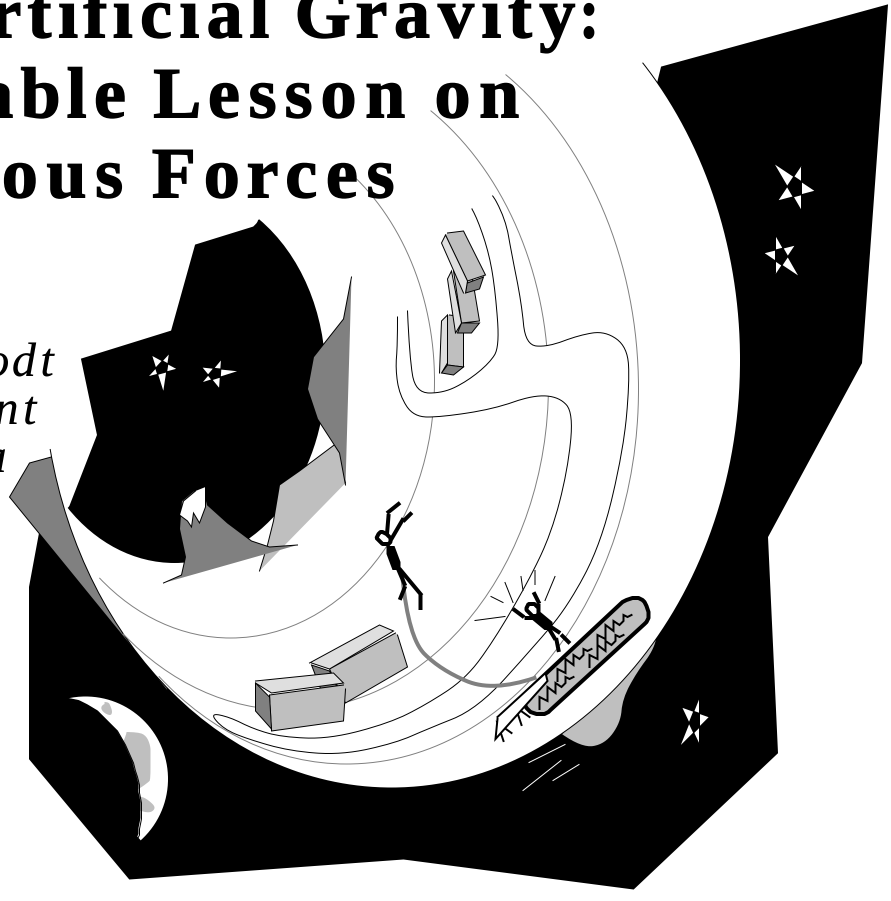
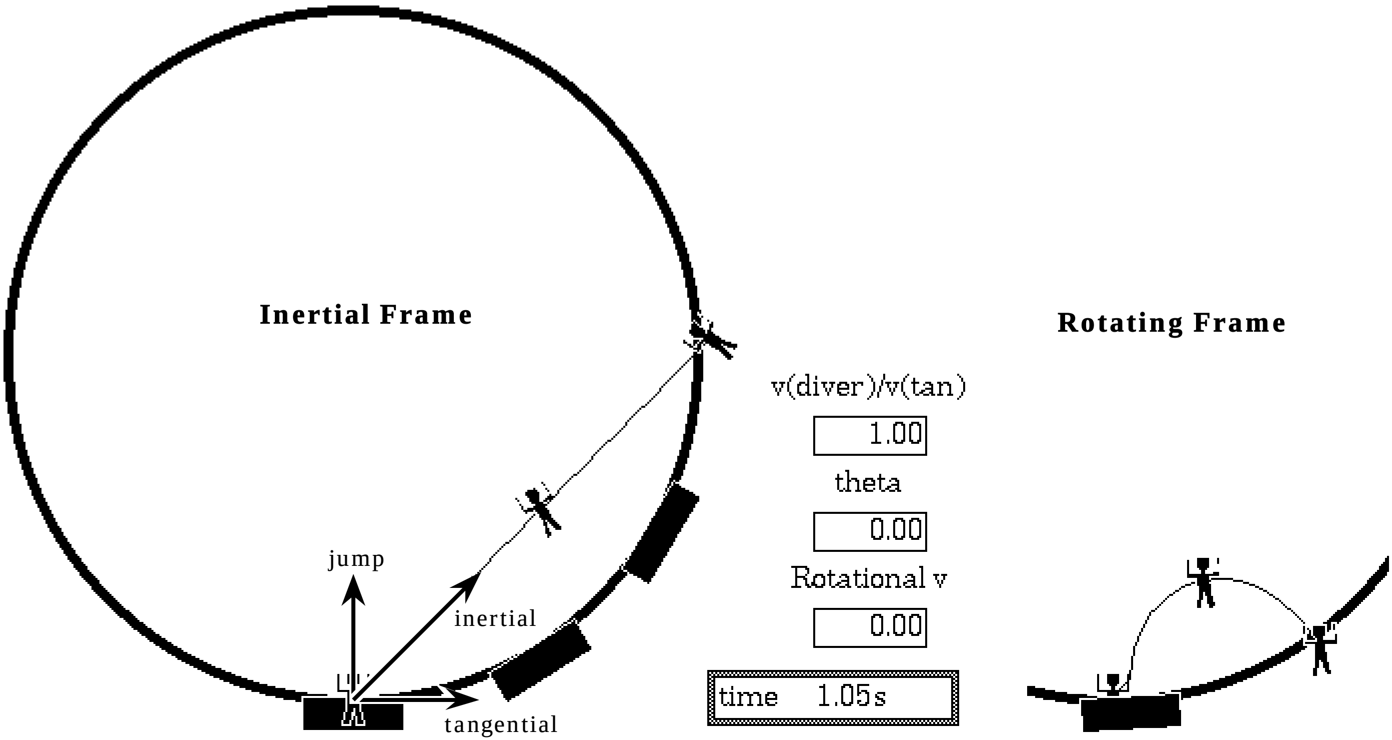


Outline

- Qualitative Considerations
  - What's different about jumping in artificial gravity?
  - How do we adjust for the Coriolis and centrifugal forces?
- Quantitative Considerations
  - What are the required conditions for a successful dive?
  - Using what we know to win a gold medal!!!
- Force and Energy in the Rotating Frame
  - The artificial gravitational potential energy.
  - The workless Coriolis force.
  - "Cusp-point trajectories."
- Conclusions

An Unsuccessful Dive



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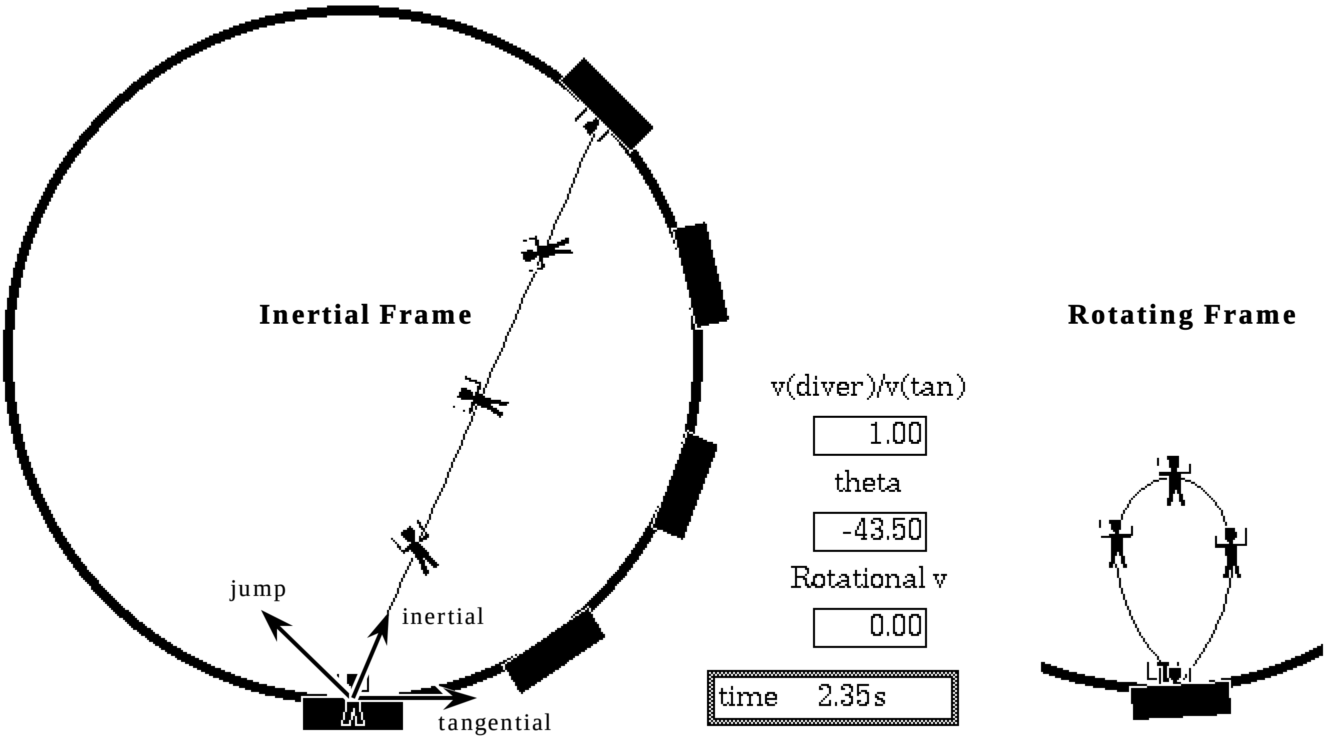
A Successful Dive



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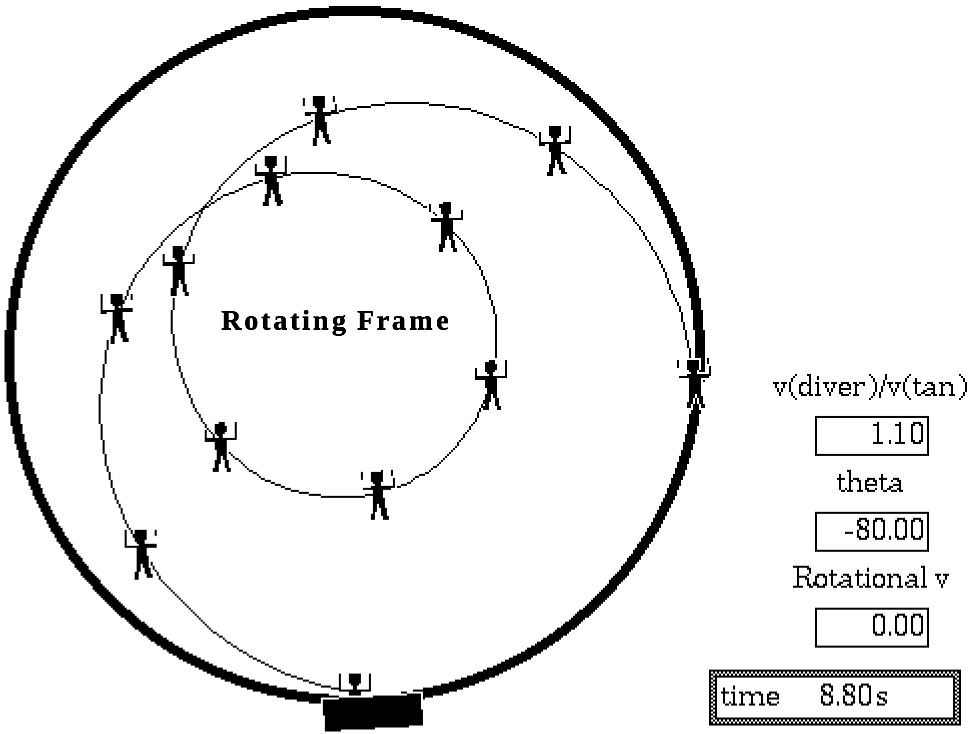
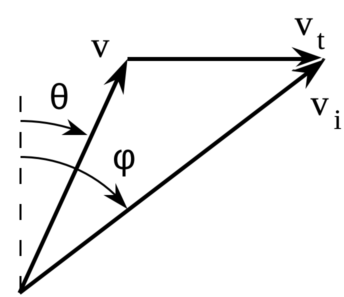
A Noteworthy, but Unsuccessful Dive



The Physics

(Relative velocity) $v^2 = v_i^2 + v_t^2 - 2\, v_i v_t \sin\varphi$ *and* $\tan\theta = \dfrac{v_i \sin\varphi - v_t}{v_i \cos\varphi}$

where $v_t = \Omega R$. Now let $u = \dfrac{v}{v_t}$ and $u_i = \dfrac{v_i}{v_t}$



$$u^2 = u_i^2 - 2\, u_i \sin\varphi + 1$$

$$\tan\theta = \frac{\sin\varphi - 1/u_i}{\cos\varphi}$$

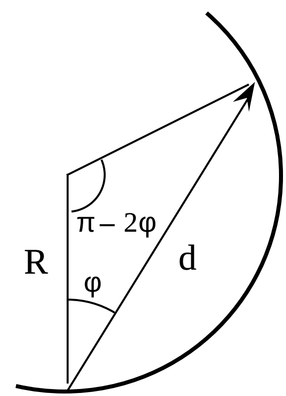
(Distance traveled) $d = 2\, R \cos\varphi$

(Time on path) $t = \dfrac{d}{v_i}$

(Success condition) $\Omega\, t = (\pi - 2\varphi) + 2n\pi, \quad (n = 0, 1, 2, \ldots)$

$$\frac{v_t}{R}\,\frac{2\, R\cos\varphi}{v_i} = (2n + 1)\pi - 2\varphi$$

$$\frac{\cos\varphi}{u_i} + \varphi = (n + 1/2)\pi$$



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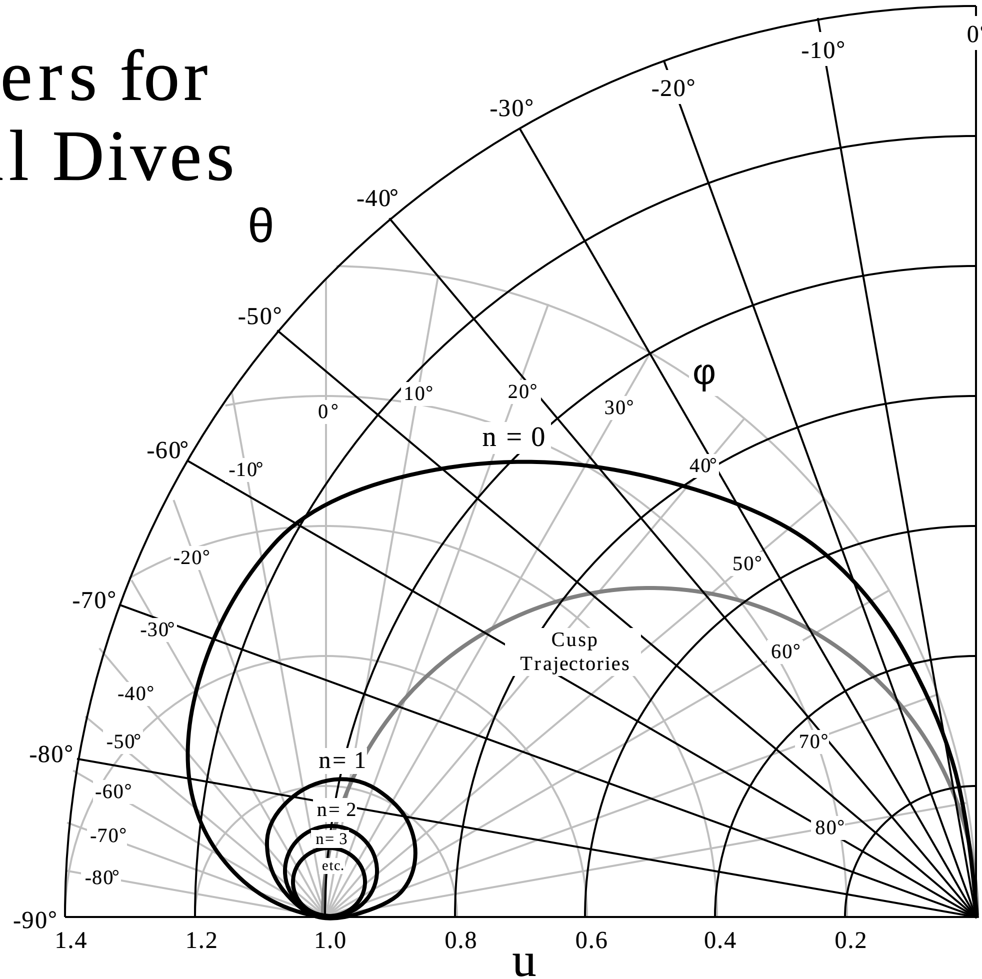
Parameters for Successful Dives



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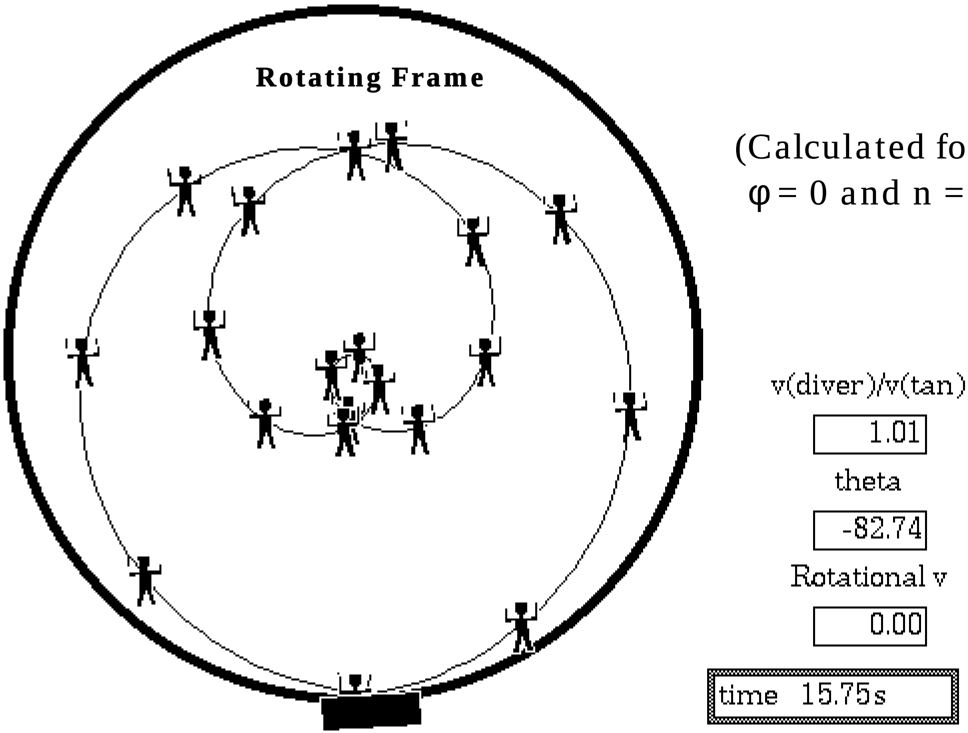
Better Diving Through Physics



(Calculated for
$\varphi = 0$ and $n = 2$)

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Force and Energy in the Rotating Frame

Centrifugal Force: $F_{cent} = mr\Omega^2$, "outward"

Centrifugal ("Artificial Gravitational") PE: $U_{cent} = -\frac{1}{2} mr^2\Omega^2$

Coriolis Force: $F_{cor} = 2m\Omega v$, "to the right"

- (Coriolis force does no work and, therefore, does not alter the total energy.)

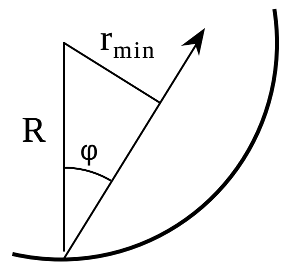
Total energy: $E = \frac{1}{2}mv^2 - \frac{1}{2} mR^2\Omega^2$

- $E > 0$ $v > v_t$ (i.e., $u > 1$) and diver *can* reach axis of rotation ($r = 0$).
- $E < 0$ $v < v_t$ (i.e., $u < 1$) and there is a minimum accessible value of r.

$$r_{min} = \sqrt{R^2 - \left(\frac{v}{\Omega}\right)^2} \qquad \frac{r_{min}}{R} = \sqrt{1 - u^2}$$

- The diver may or may *not* reach r_{min}. When r_{min} *is* realized, the trajectory in the inertial frame looks as shown at right.

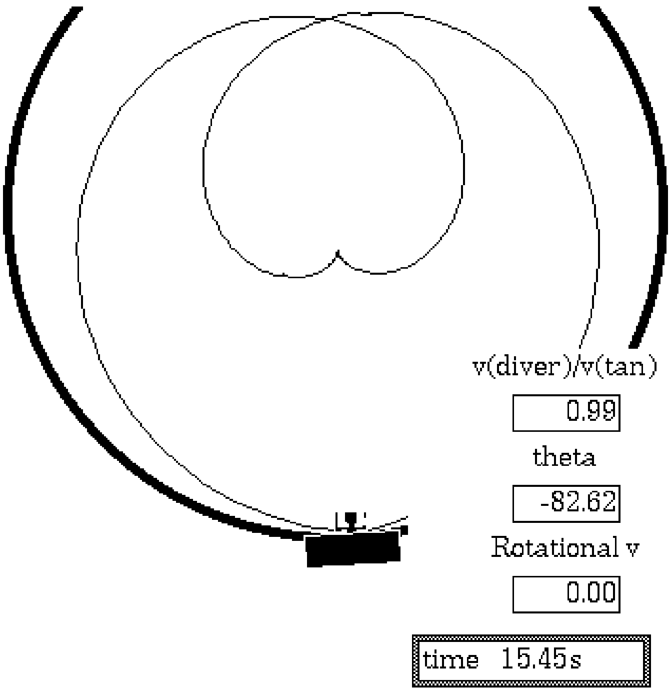
$$\frac{r_{min}}{R} = \sin\varphi \qquad u = \cos\varphi, \text{ (and } u_i = \sin\varphi)$$



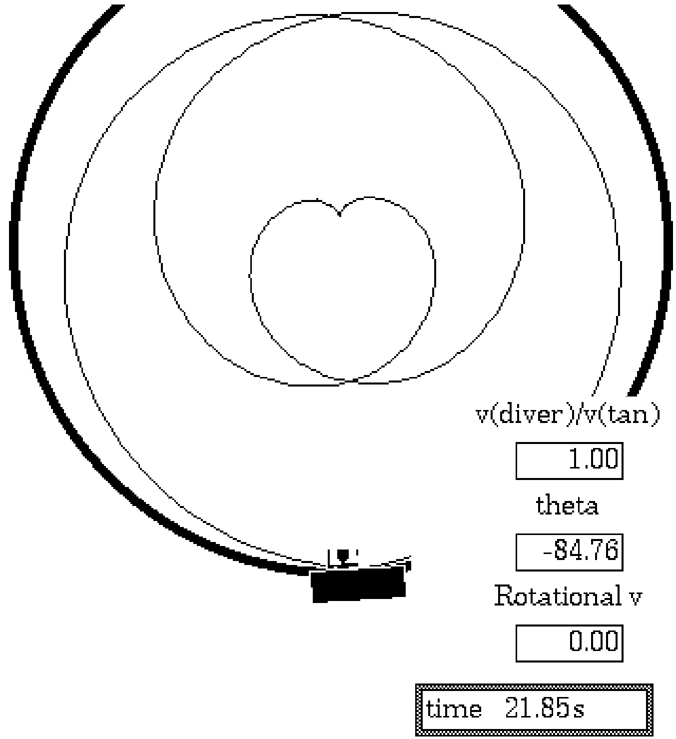
- This yields trajectories with "cusp-points"—points at which the diver is momentarily stationary in the rotating frame. (The motion of the diver near r_{min} is much like the motion of a charged particle in crossed **E** and **B** fields.)

Cusp-Point Trajectories

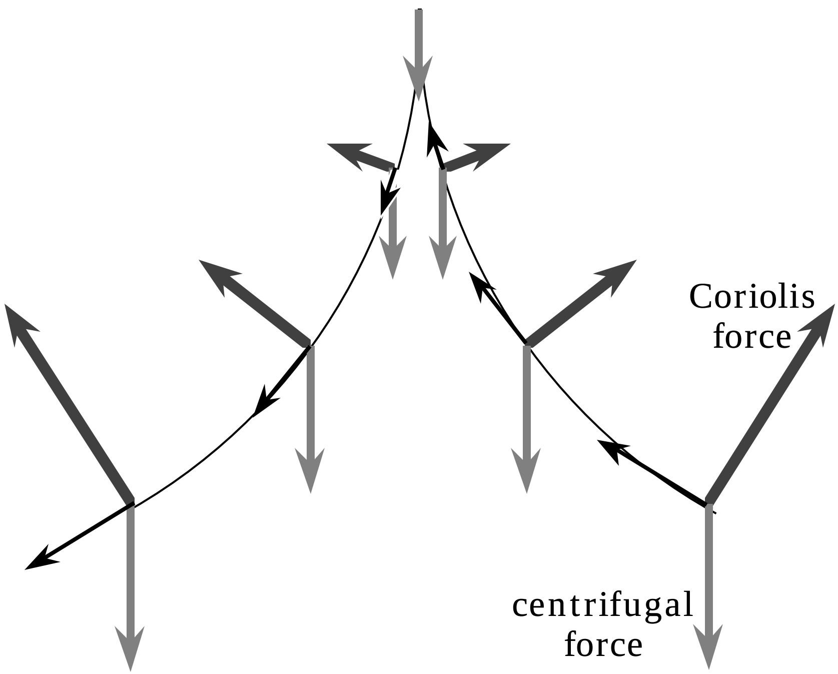
n = 2



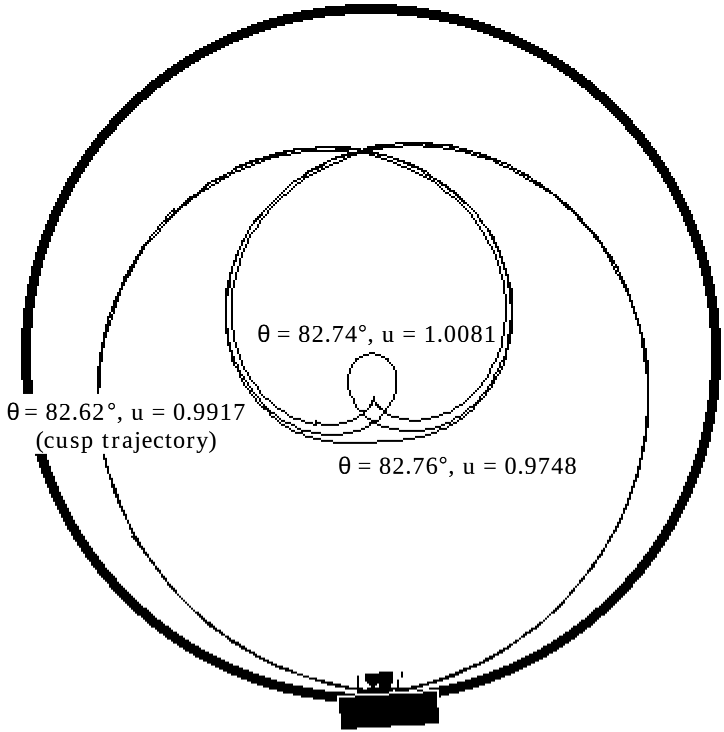
n = 3



Fictitious Forces Around the Cusp



Successful n = 2 Dives Near Cusp



Conclusions

- Straight line trajectories in the inertial frame are transformed into exotic dives in the rotating frame.
- One must *always* jump at least slightly backward to reduce one's inertial speed below the tangential speed of the frame (or "to accommodate the Coriolis force.")
- With the exception of simple dives that approximate those in normal gravity ($n = 0$, $\theta \approx 0$), successful dives begin by essentially nullifying the pre-existing tangential velocity.
- One can use energy arguments to deal with the effects of the Coriolis and centrifugal forces in the rotating frame.
- Cusp-point trajectories result from a delicate balance between the effects of the Coriolis and centrifugal forces and are reminiscent of the motion of charged particles in crossed **E** and **B** fields.

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