Appendix B: Units

Pure numbers are good for counting, but most quantities of interest in science have units (e.g., “kg,” “m,” “m/s,” etc.) The value of such a quantity cannot be understood without the units. It is meaningless to say, “The length of the table is 5,” or, “It took me 3 to finish my homework.” Admittedly, statements like, “I weigh 160,” and, “That car was doing 75,” will be understood by most people because—at least in the United States—they will automatically supply the missing units, “pounds” and “mph,” respectively. In reporting data, however, you must *always* specify the units you are using.

Units are themselves *mathematical* entities that obey the same algebraic rules that numbers do. While it is true that “100 cm = 1 m,” it is certainly *not* true that “100 = 1” or that “cm = m.” Think of “100 cm = 1 m” as *literally* meaning “100 times centimeter equals 1 times meter.” Therefore, we can also write “ $\frac{\text{m}}{\text{cm}} = 100$ ” or “ $\frac{100\text{ cm}}{\text{m}} = 1$.”

Often, after performing a calculation, you will find that the answer is expressed in unconventional and/or inconvenient units. Consider the following calculation:

$$t = \sqrt{\frac{4.57\text{ km}}{980\text{ cm/s}^2}} = \sqrt{4.663 \text{ x } 10^{-3}\frac{\text{km s}^2}{\text{cm}}} = 6.83 \text{ x } 10^{-2}\text{ s}\sqrt{\frac{\text{km}}{\text{cm}}}$$

Since we have properly operated on the units as algebraic entities, this is a *completely* valid answer for the time *t*, but it *is* expressed in rather unconventional units.

Converting Units: The “Multiply by One in the Form of ” Method

What if we want to express the time calculated above in more conventional units like “seconds”? Converting units is *always* easily accomplished using the “multiply by one in the form of ” method. We need only remember that “1 km = 1000 m” and “1 m = 100 cm.” Therefore,

$$\mathbb{1} = \frac{1000\text{ m}}{\text{km}} \quad \text{and} \quad \mathbb{1} = \frac{100\text{ cm}}{\text{m}}$$

Of course, we can raise $\mathbb{1}$ to any power and still have $\mathbb{1}$; so, for instance

$$\mathbb{1} = \sqrt{\frac{1000\text{ m}}{\text{km}}} \quad \text{and} \quad \mathbb{1} = \sqrt{\frac{100\text{ cm}}{\text{m}}}$$

Now we simply write

$$6.83 \text{ x } 10^{-2}\text{ s}\sqrt{\frac{\text{km}}{\text{cm}}} = 6.83 \text{ x } 10^{-2}\text{ s}\sqrt{\frac{\text{km}}{\text{cm}}} \times \underbrace{\sqrt{\frac{1000\text{ m}}{\text{km}}}}_{\text{“one in the form of”}} \times \underbrace{\sqrt{\frac{100\text{ cm}}{\text{m}}}}_{\text{“one in the form of”}} \quad 21.6\text{ s}$$

Please note the following:

- We have multiplied the right side of the equation by two factors of 1 so we have *not* changed the *value* of the quantity; we have only *converted* the *units* in which it is expressed. This is why our "one in the form of " factors are called "unit conversion factors."
- We have chosen the particular conversion factors based on the conversion we wish to accomplish. In general, insert conversion factors that will algebraically cancel the units you are trying to convert.

As a final example, suppose that we actually wanted to express the time *t* in hours. Since "1 min = 60 s" and "1 hr = 60 min," we would write

$$21.6\text{ s} = 21.6\text{ s} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{1\text{ hr}}{60\text{ min}} = 6.00 \text{ x } 10^{-3}\text{ hr}$$

Exercises:

1. Use the "multiply by one in the form of " method to express...

a) 79.3 cm in m.

b) 0.0450 kg in g.

c) 3.22 min in s.

d) 49 inches in cm.

e) $23\ \frac{\text{cm}}{\text{s}}$ in $\frac{\text{miles}}{\text{hr}}$.

f) $35.6 \text{ x } 10^{-2}\ \frac{\text{g cm}}{\text{s hr}}$ in N.

g) $3.44 \text{ x } 10^{3}\ \frac{\text{furlongs}}{\text{fortnight}}$ in $\frac{\text{cm}}{\text{min}}$.

h) $2.77\ \frac{\text{BTU}}{\text{horsepower}}$ in days.

2. Why is it *impossible* to "convert" pounds to kg? ...m^2 to liters? ...etc.