

Problem #1—Nonlinear Iterative Maps and the Period Doubling Route to Chaos

Exploring the behavior of simple nonlinear iterative functions like the logistic map, given by

$$x_{i+1} = r x_i (1 - x_i) \quad (\text{with } 0 < r < 4)$$

can serve as an instructive introduction to some of the features of nonlinear dynamics. We find that, for some values of “the control parameter” r , the generated sequence of iterates converges to a single stable value. For others, it converges to a repeating sequence of values. Most surprisingly, for some values of r , the sequence never repeats—an example of deterministic chaotic behavior. One very characteristic observed behavior is called “the period doubling route to chaos” in which, as r increases, the sequence of iterates repeatedly “bifurcates” from a single stable value, to two repeating values, to four repeating values, to eight, sixteen, etc., with the bifurcations coming more and more “frequently”—i.e., over smaller and smaller changes in the control parameter r . Eventually we reach a critical value of r beyond which chaotic behavior (i.e., nonrepeating or “infinite period” behavior) is observed.

Your assignment is to

1. Write a program that will take two different “seed values” (i.e., values for x_1) and graph the (two) resulting sequences as functions of the iteration index i . You should think about how best to present your graphs. For instance, should the user be able to determine the ranges of both axes or should the ranges be fixed or should the program somehow determine the best ranges? Should the graph perhaps overwrite itself, starting over when the screen is filled? Should it go for a set number of iterations or until the user presses a key?
2. Use your program to look for those values of r at which bifurcations take place, to observe the chaotic behavior that generally occurs for larger values of r , and to look for “regions of stability” in the otherwise generally chaotic range of r .
3. Use your program’s ability to plot the sequences for two different seeds to see what happens when the seeds are very close or very far away. You might want to add a feature that allows you to plot the *difference* in the two sequence values as a function of the iteration index.
4. (Optional) Try replacing the mapping function given above with and see what happens.

$$x_{n+1} = r \sin(x_n) \quad (\text{with } 0 < r < 1)$$