Assignment: From Problem Set #10 (Serway and Beichner) : 26, 36, 39, 44, 53, 54, 60, 63

10-26 (S&B 22-26) The maximum theoretical value is

$$Q_h/W = Q_h/(Q_h - Q_c) = T_h/(T_h - T_c) = 293/25 = 11.7$$

so the actual ratio is 1.17. Thus, each Joule of work results in 1.17 J of energy heat entering the room in the form of heat.

10-36 (S&B 22-36) First find the equilibrium temperature by solving the calorimetry problem:

$$\Delta E = \Delta E_w + \Delta E_{Ir} = m_w c_w (T_{eq} - T_w) + m_{Ir} c_{Ir} (T_{eq} - T_{Ir}) = 0$$

$$T_{eq} = \frac{m_w c_w T_w + m_{Ir} c_{Ir} T_{Ir}}{m_w c_w + m_{Ir} c_{Ir}} = 33.2°\text{C}$$

Now find the change in entropy by considering a process in which heat is quasistatically added to the water and removed from the iron until both are at the equilibrium temperature.

$$\Delta S = \oint_{T_w}^{T_{eq}} \frac{dQ_w}{T} + \oint_{T_{Ir}}^{T_{eq}} \frac{dQ_{Ir}}{T} = \oint_{T_w}^{T_{eq}} \frac{m_w c_w dT}{T} + \oint_{T_{Ir}}^{T_{eq}} \frac{m_{Ir} c_{Ir} dT}{T}$$

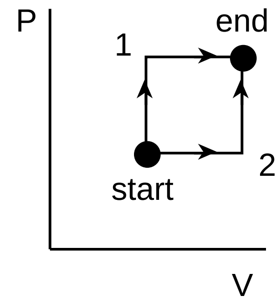
$$= m_w c_w \ln(T_{eq}/T_w) + m_{Ir} c_{Ir} \ln(T_{eq}/T_{Ir}) \qquad \text{(T in K!!)}$$

$$= +1319\ \text{J/K} - 602\ \text{J/K} = +717\ \text{J/K}$$

10-39 (S&B 22-39) Since the gas does no work and no heat is added, its energy does not change. If we assume that it behaves like an ideal gas, its temperature also does not change. Therefore, we can find the entropy change by considering a quasistatic, isothermal expansion to the same final state. In an isothermal expansion, $dQ = dW = pdV = nRT\ dV/V$ so

$$\Delta S = \oint \frac{dQ}{T} = \oint_{V_i}^{2V_i} \frac{nR\ dV}{V} = nR \ln 2 = +5.76\ \text{J/K}$$

10-44 (S&B 22-44) Two possible paths from the initial state to the final state are shown in the drawing at right.



Along path 1 we have

$$\Delta S = \oint_{\text{const V}} \frac{dQ}{T} + \oint_{\text{const P}} \frac{dQ}{T}$$

$$= nC_v \ln(T_{corner}/T_{start}) + nC_p \ln(T_{end}/T_{corner})$$

But $T_{corner}/T_{start} = P_{corner}/P_{start} = 2$

and $T_{end}/T_{corner} = V_{end}/V_{corner} = 2$ so

$$\Delta S = n(C_v + C_p) \ln 2 = n\,(6R) \ln(2) = 34.6 \text{ J/K}$$

Along path 2 we have

$$\Delta S = \oint_{\text{const P}} \frac{dQ}{T} + \oint_{\text{const V}} \frac{dQ}{T}$$

$$= nC_p \ln(T_{corner}/T_{start}) + nC_v \ln(T_{end}/T_{corner})$$

But $T_{corner}/T_{start} = V_{corner}/V_{start} = 2$

and $T_{end}/T_{corner} = P_{end}/P_{corner} = 2$ so

$$\Delta S = n(C_v + C_p) \ln 2 = n\,(6R) \ln 2 = 34.6 \text{ J/K}$$

It makes no difference which path we use because entropy is a state function, the *change* in entropy depends *only* on the initial and final states.

10-53 (S&B 22-53) For this cycle, heat is added during the isovolumetric heating and during the isothermal expansion. Since the heat added during the isothermal process is equal to the work done, we have

$$Q_{in} = nC_v\Delta T + nRT \ln(V_f/V_i)$$

$$= \frac{3}{2}\, nR(2T_i) + nR(3T_i) \ln(2)$$

$$= nRT_i\,(3 + 3 \ln 2)$$

Heat is removed during the isovolumetric cooling and during the isothermal compression. Since the heat removed during the isothermal process is equal to the *magnitude* of the work done, we have

$$Q_{out} = \text{ï}\, nC_v\Delta T\text{ï} + \text{ï}\, nRT \ln(V_f/V_i)\text{ï}$$

$$= \frac{3}{2}\, nR(2T_i) + nR(T_i) \ln(2)$$

$$= nRT_i\,(3 + \ln 2)$$

a) Thus, the net energy transferred to the gas by heat in one cycle is

$$Q_{cycle} = Q_{in} - Q_{out} = (2 \ln 2)\, nRT_i$$

b) This is also equal to the work done by the gas in one cycle so

$$e = W/Q_{in} = \frac{(2 \ln 2)}{3(1 + \ln 2)} = 0.273$$

10-54 (S&B 22-54) To take 30 g of water at 22°C and make ice at –10°C requires the removal of heat

$$Q = mc_w\, 22°C + mL_{fw} + mc_{ice} 10°C = 2763 \text{ J} + 9990 \text{ J} + 1254 \text{ J} = 14.01 \text{ kJ}$$

Since COP = Q_c/W = 3.00, removing this much heat will require 14.01 kJ/3 = 4.67 kJ of work. Doing this much work in a minute represents a power input of

$$P = W/t = 4.46 \text{ kJ}/60 \text{ s} = 77.8 \text{ W}$$

The maximum COP for this situation is $Q_c/W = T_c/(T_h - T_c) = 6.02$

10-60 (S&B 22-60) a) Work is done *by* the gas during the isothermal expansion and *on* the gas during the isobaric compression. The net work done by the gas in one cycle is

$$W = nRT \ln(V_f/V_i) + P(V_f - V_i)$$

$$= P_A V_A \ln(5) + P_B (V_C - V_B)$$

$$= (50 \ln(5) - 40) \text{ L atm} = 4.10 \text{ kJ}$$

c) Heat energy is expelled by the gas along the isobaric compression

$$Q_{out} = |Q| = |nC_p \Delta T| = \frac{5}{2} nR|\Delta T| = \frac{5}{2} P|\Delta V| = 100 \text{ L atm} = 10.1 \text{ kJ}$$

b) The heat in is just the sum of the work done and the heat expelled, so

$$Q_{in} = W + Q_{out} = 14.2 \text{ kJ}$$

d) $e = W/Q_{in} = 0.288$

10-63 (S&B 22-63) a) Heat is added along paths AB and BC so

$$Q_{in} = nC_v(T_B - T_A) + nC_p(T_C - T_B)$$

$$= \frac{3}{2} nR(T_B - T_A) + \frac{5}{2} nR(T_C - T_B)$$

$$= \frac{3}{2}(P_B V_B - P_A V_A) + \frac{5}{2}(P_C V_C - P_B V_B)$$

$$= \frac{3}{2}(2P_i V_i) + \frac{5}{2}(3P_i V_i)$$

$$= 10.5\, P_i V_i = 10.5\, nRT_i$$

b) Heat is removed along paths CD and DA so

$$Q_{out} = |nC_v(T_D - T_C) + nC_p(T_A - T_D)|$$

$$= \left|\frac{3}{2} nR(T_D - T_C) + \frac{5}{2} nR(T_A - T_D)\right|$$

$$= \left|\frac{3}{2}(P_D V_D - P_C V_C) + \frac{5}{2}(P_A V_A - P_D V_D)\right|$$

$$= \frac{3}{2}(4P_i V_i) + \frac{5}{2}(1P_i V_i)$$

$$= 8.5\, P_i V_i = 8.5\, nRT_i$$

c) $e = W/Q_{in} = (Q_{in} - Q_{out})/Q_{in} = 0.190$

d) The extreme temperature ratio is $T_C/T_A = P_C V_C / P_A V_A = 6$ so the efficiency of a Carnot engine running between these extremes is

$$e_C = 1 - T_c/T_h = 1 - 1/6 = 0.833$$