# HW #5 Solutions

Professor Mallinckrodt

**Assignment:** From Problem Set #5: 3, 4, 5.1, 9, 11 & From Problem Set #6: 1, 3, 6, 7, 11

Please note that my solutions would probably rate something like a low 3 due to the minimal amount of explanation and the relative lack of figures. They are intended to help *you* work through problems that you may have had trouble with, *not* to demonstrate superior quality writeups. *Please* come see me if you have further questions. Thanks!

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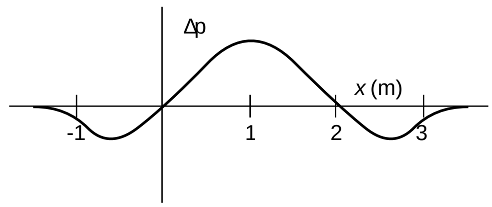
**5-3** a) Reading directly from the graph. We see that molecules are displaced furthest to the left (*i.e.* have the most negative values for s) when x = 2.0 m.

b) It means that the displacements are *very* small!

c) It means that particles have squeezed in toward those at x = 1 m (which have not moved).

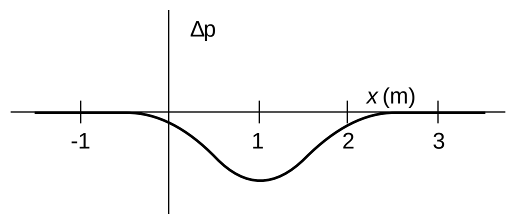
d) As a result, the density and pressure increase near x = 1 m.

e) Particles near x = 0 and 2 are displaced *along with* their neighbors leading to no pressure change at those locations. For x between –1 m and 0 m and for x between 2 m and 3 m, neighbors to each side are displaced in such a way to end up *farther* away at the end and, thus, produce negative pressure changes. The result is something like that shown at right.




f) There is nothing in the snapshot that would indicate the direction of the wave.

g) For the second plot, molecules are maximally displaced to the right at large positive x and to the left at large negative x. Thus, the pressure is relatively small in the region where the displacements are changing and are smallest where they change fastest at x = 1 m. The result is a pressure profile something like that shown at the right.

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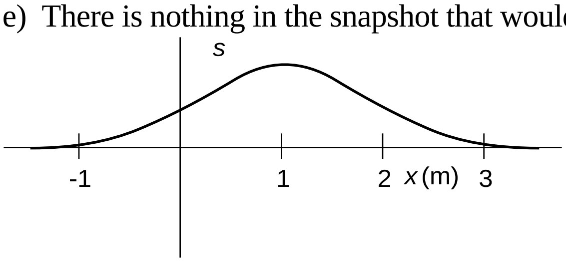
**5-4** a) It means that the pressure there is *less* than it was before the wave was present.

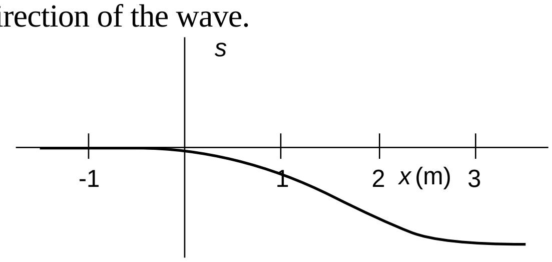
b) p(x = 0 m) ≈ p(x = 1 m) ≈ p(x = 2 m) ≈ $1.013 \times 10^5$ Pa. The pressure changes associated with the wave are exceedingly small.

c) The molecules would have to be moved farther apart. So molecules at small positive x are moved to the right and vice-versa.

d) Beginning around x = –1 m, the pressure change becomes negative meaning that molecules must be moved progressively further to the right. At x = 1 m there is no pressure change so particles must be moved the same amount on either side. After that the pressure change becomes positive meaning that molecules must be moved progressively *less* far to the right. The result is as shown in the figure below left.

e) There is nothing in the snapshot that would indicate the direction of the wave.




f) For the second plot, the pressure change is always positive so, as we move toward the right, molecules must be progressively moved farther and farther to the left as shown in the figure above right.

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**5-5.1** Taking the derivatives of the two graphs for s(x) shown in problem 3 and inverting them, does indeed give graphs that look like those already shown in its solution.

**5-9** a) 110 dB $I = I_o \times 10^{11} = 0.100\ \text{W/m}^2$ so $P = I(4\pi r^2) =$ 5.03 W

b) Five watts doesn't seem like very much power to be causing a *very* loud sound. (The explanation is that speakers tend to be *very* inefficient “electro-acoustic transformers”, that is, they are not very efficient at taking the electrical power output of an amplifier and changing it into acoustical power.)

c) $P = IA = 0.100\ \text{W/m}^2 \times 1.0 \times 10^{-4}\ \text{m}^2 =$ 10 μW

d) $I = P/(4\pi r^2)$ so twice as far 1/4 the intensity change in sound level of 10 log(1/4) dB = -6.0 dB so the new sound level would be 104 dB.

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**5-11** We have

$$\text{fractional change in f} \equiv \frac{f_f - f_i}{f_i}$$

$$-0.150 = \frac{f_{\text{receding train}}}{f_{\text{approaching train}}} - 1$$

Using the Doppler formula for a moving source, we have

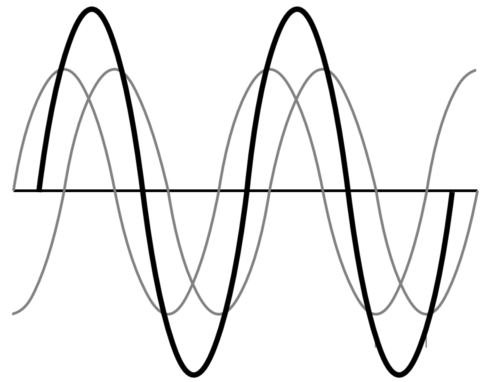
$$-0.150 = \frac{f_{\text{source}}\left(\dfrac{v_{\text{sound}}}{v_{\text{sound}} + v_{\text{source}}}\right)}{f_{\text{source}}\left(\dfrac{v_{\text{sound}}}{v_{\text{sound}} - v_{\text{source}}}\right)} - 1$$

$$-0.150 = \frac{v_{\text{sound}} - v_{\text{train}}}{v_{\text{sound}} + v_{\text{train}}} - 1$$

and solving for $v_{\text{train}}$,we get

$$v_{\text{train}} = \frac{0.15}{1.85}\ v_{\text{sound}} = \underline{27.8\ \text{m/s}}$$

**6-1** a) When two waves of amplitude A that are out of phase by an amount φ are added, the superposition result is a wave with an amplitude of 2Acos(φ/2) and a phase difference of φ/2 relative to either one. The result here would be an amplitude of 2 cos(45°) = $\sqrt{2}$ times that of either one and with a phase difference of 45° relative to either one as shown at right.

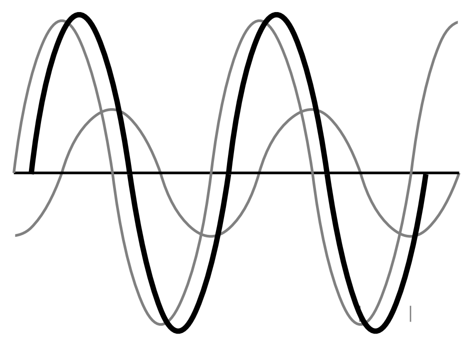


b) For other phase differences we'd have

| phase diff | amplitude | relative phase |
|---|---|---|
| 45° | 1.85 A | 22.5° |
| 90° | 1.41 A | 45° |
| 135° | 0.765 A | 67.5° |
| 180° | 0 | n/a |
| 270° | 1.41A | 135° |

(Note that when the "phase difference" φ is greater than 180°, we are really talking about a relative phase difference of 360° – φ. So that the listing above for 270° is really the same as that for 90°.)

c) If one wave has a larger amplitude, the result will be a wave that is closer in phase and amplitude to that of the large wave. In the specific case mentioned, the result might look something more like that shown at right.



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**6-3** a) For the sounds to add in phase, the path difference must be an integer number of wavelengths with λ = v/f = 0.429 m.

Note that at x = 0, the path difference is 3.0 m = 6.997 λ and that as we go out on the x-axis, the path difference decreases. Thus the first position at which the waves add in phase is the place where

$$\sqrt{x^2 + (3\text{ m})^2} - x = 6\lambda \qquad x = \underline{0.463\text{ m}}$$

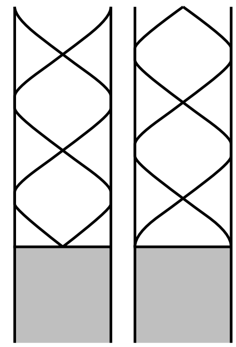
Similarly, the next two happen when

$$\sqrt{x^2 + (3\text{ m})^2} - x = 5\lambda \qquad x = \underline{1.03\text{ m}}$$

$$\sqrt{x^2 + (3\text{ m})^2} - x = 4\lambda \qquad x = \underline{1.77\text{ m}}$$

b) Clearly this sequence continues for 3λ, 2λ, and 1λ. The path difference approaches but never reaches 0. So there are <u>6 locations</u> along the positive x-axis where the sounds add in phase.

**6-6** a and b) The pressure and amplitude patterns are (as always) out of phase by 1/4 cycle. There is a displacement node at the water level and a pressure node at the open end. The results are shown at right. (Note the three pressure antinodes.)




c) This is the third harmonic of a closed-open tube which we know has a frequency of 5 times the fundamental. So the fundamental is 100 Hz and the tube will resonate at odd integer multiples of that frequency, *e.g.*, 300 Hz, 700 Hz, 900 Hz, 1100 Hz and so on.

d) The length is 5/4 $\lambda$ = (5/4) (v/f) = 85.8 cm

e) It would also resonate when L = 3/4 $\lambda$ or 1/4 $\lambda$, that is, L = 51.5 cm and 17.2 cm

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**6-7** a) Could be 328 Hz or 332 Hz. (Difference must be 2 Hz.)

b) Must be 333 Hz since the beat frequency increased!

c) We know that $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ so that $f_f/f_i = \sqrt{T_f/T_i}$ $\quad T_f/T_i = (330/333)^2$

Now, the percentage change in tension is given by

$$\text{percentage change in T} \equiv \frac{T_f - T_i}{T_i} \times 100\% = (T_f/T_i - 1) \times 100\% = -1.79\%$$

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**6-11** Using the closed-open end result for the fundamental frequency we have

$$f = \frac{v}{4L} \qquad L = \frac{v}{4f} = 2.9 \text{ cm}$$

which seems reasonable.