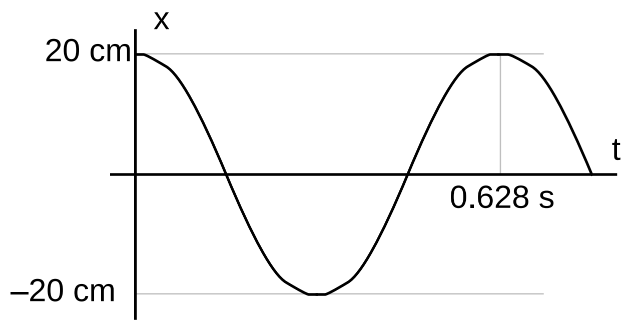
Assignment: From Problem Set #1: 3.1, 4.0, 4.1, 5, 7, 11, 12, 14

Please note that my solutions would probably rate something like a low 3 due to the minimal amount of explanation and the relative lack of figures. They are intended to help *you* work through problems that you may have had trouble with, *not* to demonstrate superior quality writeups. *Please* come see me if you have further questions. Thanks!

3.1 a) Released at rest x_i is the amplitude = 20 cm.

$T = 2\pi\sqrt{m/k} = 0.628$ s.

Therefore, the sketch is as shown at right



b) This function can be written as

$A \sin(\omega t + \varphi)$ if A = 20 cm and $\varphi = \pi/2$.

It can be written as

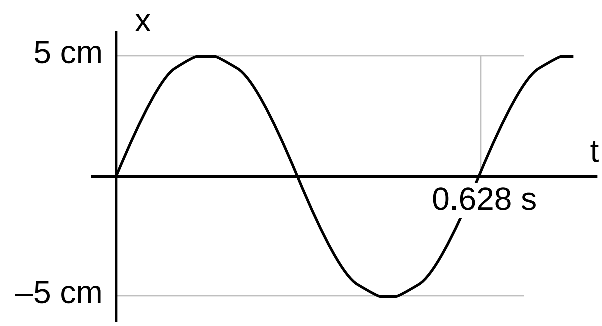
$A \cos(\omega t + \varphi)$ if A = 20 cm and $\varphi = 0$.

It can be written as

$A \sin\omega t + B \cos\omega t$ if A = 0 cm and B = 20 cm.

c) Given an initial velocity at the equilibrium position $v_{max} = 50$ cm/s. $\omega = \sqrt{k/m} = 10$/s so $A = v_{max}/\omega = 5.0$ cm.

Same period, so sketch is as shown.



This function can be written as

$A \sin(\omega t + \varphi)$ if A = 5.0 cm and $\varphi = 0$.

It can be written as

$A \cos(\omega t + \varphi)$ if A = 5.0 cm and $\varphi = -\pi/2$.

It can be written as

$A \sin\omega t + B \cos\omega t$ if A = 5.0 cm and B = 0 cm.

4.0 a) It takes 3.0 s for half a cycle so T = 6.0 s. The amplitude is half the range of motion or 3.5 m. The equilibrium position is in the middle of the range or +1.5 m. The angular frequency is $2\pi/T$ or 1.05 rad/s.

b) We see that the phase = 0 when t = 1.0 s. That is,

$$\omega(1.0\text{ s}) + \varphi = 0 \qquad \varphi = -\omega(1.0\text{ s}) = -1.05\text{ rad}$$

so

$$x(t) = 1.5\text{ m} + 3.5\text{ m}\cos[(1.05\text{ rad/s})\,t - 1.05\text{ rad}]$$

4.1 As in problem 4.0, we see *here* that A = 25 cm and x_{eq} = –5.0 cm. We also see that

$x(0.30 \text{ s}) = -5.0 \text{ cm} + 25 \text{ cm} \cos(\omega(0.30 \text{ s}) + \varphi) = -30 \text{ cm}$

$x(0.50 \text{ s}) = -5.0 \text{ cm} + 25 \text{ cm} \cos(\omega(0.50 \text{ s}) + \varphi) = 0$

These two equations can be solved for their respective phases to give

$\omega(0.30 \text{ s}) + \varphi = \arccos(-1) = \pi = 3.1416$

$\omega(0.50 \text{ s}) + \varphi = \arccos(0.20) = 1.564\,\pi = 4.9137$

(Note: *Clearly* the (cosine) phase at 0.50 s is more than 3/4 of a cycle. After all, it was half a cycle at 0.30 s. You need to be *very* careful thinking about the results of invese trig functions. *Be sure* that you know the approximate answer *before* you calculate the result so that you will be able to properly interpret what your calculator says!!)

Solving the two equations simultaneously we get

$\omega = 8.86$ rad/s and $\varphi = 0.483$ rad

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5 a) We are given $A_r = A_g = A$ and $\omega_r = 2\omega_g$. So $v_{max-r}/v_{max-g} = \omega_r A/\omega_g A = 2$. That is, the maximum speed of the red particle is twice that of the green one.

b) $a_{max-r}/a_{max-g} = \omega_r^2 A/\omega_g^2 A = 4$. That is, the maximum acceleration of the red particle is four times that of the green one.

c) First, from $a_{max} = \omega^2 A$, we find that $\omega_g = \sqrt{a_{max-g}/A}$.

Then we find $v_{max-r} = \omega_r A = 2\omega_g A = 2\sqrt{a_{max-g}\,A}$ = 2.80 m/s.

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7 First find $\omega = \sqrt{k/m}$ = 122.9 rad/s. (That's fast! But then it *is* a stiff spring *and* a small mass!) Next note that if $x(t) = A\cos(\omega t + \varphi)$ then $v(t) = -\omega A \sin(\omega t + \varphi)$. The initial conditions are

$x_i = x(0) = -12 \text{ cm} = A\cos\varphi$ and $v_i = v(0) = 27 \text{ cm/s} = -\omega A \sin\varphi$

Solving these two equations for A and φ gives us

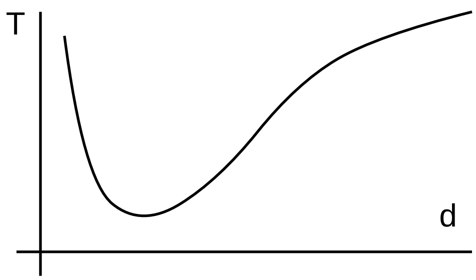
$\varphi = \arctan(-v_i/\omega x_i) = \arctan(0.0183) =$ ~~0.0183 rad~~ or 3.160 rad

$A = \sqrt{x_i^2 + (v_i/\omega)^2}$ = 12.0 cm.

(Note that we took the value of φ in the third quadrant because the initial conditions demand that $\cos\varphi < 0$ and $\sin\varphi < 0$.)

11 a) Starting with the result from the book for T and using the parallel axis theorem we find that the rotational inertia about the axis through the support is given by

$$I = I_{cm} + md^2 \text{ so that } T = 2\pi\sqrt{\frac{I_{cm} + md^2}{mgd}} \qquad \text{Q.E.D.}$$



b) The period increases without bound as $d \to 0$ and as $d \to \infty$. In particular, T is proportional to $\sqrt{1/d}$ for small d and to $\sqrt{d}$ for large d. The plot, therefore looks something like that shown at right.

c) T will minimize when $(T/2\pi)^2$ minimizes so set $\frac{d(T/2\pi)^2}{dd} = 0$ and find

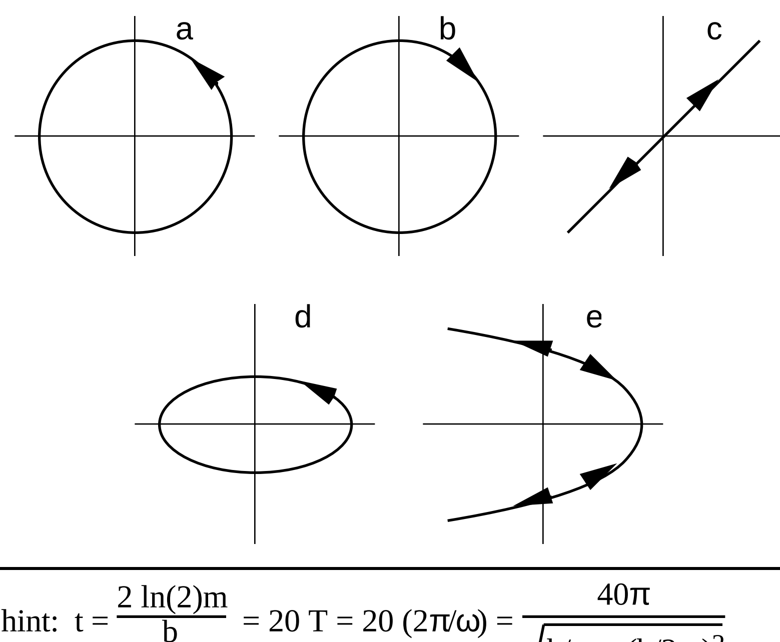
$$2md\,(mgd) - (I_{cm} + md^2)\,mg = 0 \qquad I_{cm} = md^2 \text{ or } d = \sqrt{I_{cm}/m}$$

The minimum period is, therefore,

$$T_{min} = 2\pi\sqrt{\frac{2md^2}{mgd}} = 2\pi\sqrt{\frac{2d}{g}}$$

d) This is the period of a pendulum with a length l = 2d.

12 a) Same amplitude, y peaks 1/4 cycle after x   CCW circle.
b) Same amplitude, y peaks 1/4 cycle before x   CW circle.
c) Same amplitude, x and y peak at the same time   diagonal line in first and third quadrants.
d) x amplitude is twice the y amplitude, y peaks 1/4 cycle after x   CCW ellipse elongated along the x -axis.
e) x frequency is twice the y frequency. Completes one *full* side-to-side in the time y goes from the middle to the top and back. Repeats during the bottom half-cycle



14 Using the hint: $t = \frac{2\ln(2)m}{b} = 20\,T = 20\,(2\pi/\omega) = \frac{40\pi}{\sqrt{k/m - (b/2m)^2}}$

Solving this equation for b^2 gives

$$b^2 = \frac{mk}{[\,20\pi/\ln(2)\,]^2 + 1/4} = 1.217 \times 10^{-4}\,mk$$

Now, we want to know the fractional reduction in the frequency and, eventually, to express it as a percentage reduction

$$\text{fractional reduction} = \frac{\omega_{natural} - \omega}{\omega_{natural}}$$

$$= 1 - \frac{\omega}{\omega_{natural}}$$

$$= 1 - \frac{\sqrt{k/m - (b/2m)^2}}{\sqrt{k/m}}$$

$$= 1 - \sqrt{1 - b^2/4mk}$$

$$= 1 - \sqrt{1 - 1.217 \text{ x } 10^{-4}/4}$$

$$= 1 - \sqrt{1 - 3.04 \text{ x } 10^{-5}}$$

$$= 1.5 \text{ x } 10^{-5}$$

$$= 0.0015\%$$