The Big Ideas—Chapter 14

(Serway and Beichner, Physics for Scientists and Engineers, 5th Edition)

Sections 1&2	
Every particle in the universe attracts every other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.	$F_g = \frac{G\, m_1 m_2}{r^2}$
By Newton's third law, each particle exerts the same force on the other—a result that you will notice is built into the formula due to the symmetrical treatment of both particles.	
To say that the force is "attractive" is to say that the force *on* either particle is *toward* the other.	
It can be shown that spherically symmetric objects also obey this law where *r* is the distance between their *centers* (and must be *greater* than the sum of the two diameters.)	
Since *every* part of a *small*, but *non*spherically symmetric object is about the same distance from the center of a large spherically symmetric object (like Earth), the force between them is *also* given by the same law. In this case *r* is the distance between the center of the large spherically symmetric object and *any* part of the small object. (The point is that you'll get essentially the same *r* in any event.)	
The proportionality constant *G* in the law of universal gravitation can only be determined by measuring the gravitational force between two known masses.	$G = 6.673 \times 10^{-11}\ \mathrm{N\ m^2/kg^2}$
This is difficult since the gravitational force between masses that are small enough to be independently measurable is exceedingly small.	
Section 3&6	
The gravitational field (which has the same value as the "free fall acceleration") at the surface of a large spherical body is determined by that body's mass and radius.	$g = \frac{F_g}{m} = \frac{GM}{R^2}$
More generally, the gravitational field at any altitude may be obtained by simply adding the altitude to the body's radius (i.e., to get the *total* distance to the body's center.)	

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Section 4&5

Well before Newton formulated his laws of motion and his law of universal gravitation, Kepler determined three laws of planetary motion directly from observations of the planets. These laws turn out to be precisely *predicted* by (and, therefore, special cases of) Newton's far more general laws.

Kepler's laws are:

1 All planes move in elliptical orbits with the Sun at one focal point.

(The proof of this law using Newton's laws is somewhat beyond the scope of this course.)

2 The radius vector drawn from the Sun to a planet sweeps out equal areas in equal time intervals.

(This turns out to follow directly from the fact that a planet's angular momentum about the Sun is conserved.)

3 The square of the orbital period of any planet is proportional to the cube of the semimajor axis of its elliptical orbit.

(This is easy to prove—as shown at right—for *circular* orbits using Newton's expression for the force on the planet, his second law of motion, and the kinematic expression for acceleration of an object in uniform circular motion.)

$$F = M_{\text{planet}} a$$

$$\frac{GM_{\text{sun}} M_{\text{planet}}}{r^2} = M_{\text{planet}} \left(\frac{v^2}{r} \right)$$

$$\frac{GM_{\text{sun}}}{r^2} = \frac{(2\pi r/T)^2}{r}$$

$$T^2 = \left(\frac{4\pi^2}{GM_{\text{sun}}} \right) r^3$$

Section 7

Gravity is a conservative force. Thus we can define a "potential energy" to describe the gravitational interactions between particles as they change their separation distance.

Previously we defined a gravitational potential energy function, $U = mgh$, which 1) masked the fact that two masses were interacting (the effect of the second one was buried in "g"), and 2) was only *approximately* valid and could only be used in situations in which the gravitational field did not *change* appreciably. The more general expression derived here applies in all cases.

$$U = -\frac{Gm_1 m_2}{r}$$

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Section 8 **The law of energy conservation (using the new. more general expression for gravitational potential energy) is *very* useful in analyzing the motion of satellites.** In circular motion a satellite *must* have *just the right* speed and that turns out to give it a total energy that depends primarily on its orbital radius. (Notice that the total energy for an orbiting satellite is *less* than zero. This is simply due to the fact that we found it convenient to set $U = 0$ for "infinite separation.") For a satellite to "escape", it must have enough energy to get arbitrarily far away from the "gravitating object"—i.e., the object from which it is escaping. Looking at the equation for gravitational potential energy, we see that this means its total energy must be at *least* zero. Conservation of energy can then be used to calculate the minimum required "escape velocity." (Note that $v_f = 0$ in this case and that v_{esc} depends on the mass of the gravitating object and how close it was originally.)	$K = \frac{1}{2} m v^2 = \frac{GMm}{2r}$ $E = K + U = -\frac{GMm}{2r}$ $E_o = E_f$ $-\frac{GMm_{sat}}{r} + \frac{1}{2} m_{sat} v_{esc}^2 = 0$ $v_{esc} = \sqrt{\frac{2GM}{r}}$
Section 9&10 These sections provide 1) the general expression for the gravitational force between particles and *extended* objects—objects that have their mass arbitrarily *distributed* and 2) the proof that a spherical shell a) exerts *no force* particles that are *inside* the shell and b) behaves gravitationally as if its total mass were concentrated at its center for particles that are *outside* the shell.	