

The Big Ideas—Chapter 13

(Serway and Beichner, Physics for Scientists and Engineers, 5th Edition)

<p><i>Sections 1 and 2</i></p> <p>When a particle is subject to a <i>net restoring force</i> that is purely a <i>linear function</i> of the displacement from an equilibrium position, the particle will execute <i>simple harmonic motion</i> about the equilibrium position.</p> <p>Simple harmonic motion has the following specific characteristics:</p> <ol style="list-style-type: none"> 1 The particle's displacement from the equilibrium position is a <i>sinusoidal</i> function of time. 2 The <i>period</i> of motion is determined exclusively by the <i>stiffness</i> of the restoring force and the <i>inertial mass</i> of the particle. Notably, it does <i>not</i> depend on the <i>amplitude</i> of the motion. 3 The <i>velocity amplitude</i> is equal to the angular frequency of the motion times the <i>displacement</i> amplitude (usually just called “the amplitude.”) 3 The <i>acceleration amplitude</i> is equal to the angular frequency of the motion times the velocity amplitude. <p>The amplitude and the <i>phase</i> of the sinusoidal motion are determined by the <i>initial conditions</i> of the specific motion under consideration.</p>	$F_{\text{net}} = -kx$ $x(t) = A\cos(\omega t + \phi)$ $T = 1/f = 2\pi / \omega$ $= 2\pi \sqrt{m/k}$ $v_{\text{max}} = \omega A$ $a_{\text{max}} = \omega v_{\text{max}}$
<p><i>Section 3</i></p> <p>The total energy of a simple harmonic oscillator is proportional to the square of the amplitude and it “sloshes” back and forth between potential and kinetic forms.</p> <p>Both U and K behave as squared sinusoidal functions of time that are <i>out of phase</i> with each other. That is, when $U = E$, $K = 0$ and vice-versa.</p>	$E = U + K$ $= \frac{1}{2}kA^2$ $= \frac{1}{2}mv_{\text{max}}^2$ $U = \frac{1}{2}kx^2 = E\cos^2(\omega t + \phi)$ $K = \frac{1}{2}mv_{\text{max}}^2 = E\sin^2(\omega t + \phi)$

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<p><i>Section 4</i></p> <p>The <i>pendulum</i> is a device that behaves approximately as a simple harmonic oscillator for <i>small amplitudes only</i>. Its period depends only on the distance from the point of suspension to the center of mass, the local gravitational field strength, and the way the mass is distributed but—notably—not on the mass <i>itself</i>.</p> <p>The most general type of pendulum is called a <i>physical pendulum</i>. (Note that I is the rotational inertia of the pendulum about an axis through the chosen point of suspension. Note also that the ratio m/I is a function <i>only</i> of the way the mass is distributed but <i>not</i> the mass <i>itself</i>.)</p> <p>When the mass is concentrated at one point, we call it a <i>simple pendulum</i>. In that case the “distance from the point of suspension to the center of mass” is simply the <i>length</i> of the pendulum and the period depends <i>only</i> on that length and the local gravitational field strength.</p> <p>The <i>torsional pendulum</i> is a <i>rotating simple harmonic oscillator</i> created by the action of a restoring <i>torque</i>. Its period depends on the <i>torsion constant</i> and the rotational inertia of the body about the axis of rotation.</p> <p>(Note that the torsional pendulum is not <i>really</i> a pendulum at all since it does not depend on gravity for its operation.)</p>	$= \sqrt{\frac{mgd}{I}}$ $\text{simplependulum} = \sqrt{\frac{g}{L}}$ $= \sqrt{\frac{\tau}{I}}$
<p><i>Section 5</i></p> <p>The motion of a simple harmonic oscillator is the <i>same</i> as the projection on one axis of an object moving (with an angular velocity equal to the angular frequency of the simple harmonic oscillator) around a circle (having a radius equal to the amplitude of the simple harmonic oscillator.)</p>	

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<p><i>Section 6</i></p> <p>When a simple harmonic oscillator is subject to a drag force, it executes damped motion which may be characterized as <i>underdamped</i>, <i>overdamped</i>, or <i>critically damped</i> depending on whether or not the damped motion is still oscillatory.</p> <p>Mathematical solutions are (relatively) easily obtained when the drag force is a <i>linear</i> function of the velocity.</p> <p>In the <i>underdamped</i> case ($b/2m < \sqrt{k/m}$), the amplitude of the oscillations decays exponentially and the frequency is slightly reduced from its <i>undamped</i> value.</p>	$F_{\text{net}} = -kx - b v$ $x(t) = A e^{-\frac{b}{2m}t} \cos(\omega_d t + \phi)$ $= \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$ <p>where $\omega_0 = \sqrt{k/m}$</p>
<p><i>Section 7</i></p> <p>When a simple harmonic oscillator is subject to both a linear drag force and an external sinusoidal drive force, it settles into a steady state motion that is sinusoidal at the same frequency as the drive force.</p> <p>The amplitude of the steady state motion depends sensitively on the drive frequency and reaches a maximum value very near the <i>natural</i> (undamped) frequency of the oscillator which is also often called the <i>resonance frequency</i>.</p> <p>The smaller the damping coefficient, the more sharply peaked is the so-called amplitude response curve which plots the amplitude as a function of drive frequency. Lightly damped oscillators are often referred to as “high quality” or “high Q.”</p>	$F_{\text{net}} = F_{\text{drive}} \cos(\omega_{\text{drive}} t) - kx - bv$ $x(t) = A \cos(\omega_{\text{drive}} t + \phi)$ $A = \frac{F_{\text{drive}}/m}{\sqrt{(\omega_0^2 - \omega_{\text{drive}}^2)^2 + (b/m)^2}}$