Physics 409 **Problem #2—Orbital Motion under Central Forces** Fall 2002

Consider a small satellite of mass *m* orbiting a planet in a strange quadrant of the universe where gravity has been altered to obey

$$\mathbf{F} = -k\, r^n\, \hat{\mathbf{r}}$$

where *k* may or may not depend on the masses of the two bodies and with *n not necessarily equal to –2*. (Shudder! What kind of a crazy quadrant is this? Beam me up Scotty!) Newton's second law can then be cast into the dimensionless form

$$\dot{\mathbf{v}} = -r^{n-1}\,\mathbf{r} \qquad \text{and} \qquad \dot{\mathbf{r}} = \mathbf{v}$$

where

1) the dimensionless displacement from the origin $\mathbf{r} = \dfrac{\text{the physical } \mathbf{r}}{r_o}$,

2) r_o is the initial (physical) displacement from the origin,

3) the dimensionless velocity $\mathbf{v} = \dfrac{\text{the physical } \mathbf{v}}{v_c}$,

4) v_c is the "circular orbit velocity" $= \sqrt{k r_o^{n+1}/m}$,

and

5) the dimensionless time $t = \dfrac{v_c(\text{the physical } t)}{r_o}$.

Your assignment is to:

1. Write a program that solves the differential equation of motion for the satellite using the Euler method and that displays the orbit graphically. The user should be able to input choices of values for *n*, v_{xo}, v_{yo}, and time step. (The initial position can be set to $x_o = 1$, $y_o = 0$ without loss of generality.) Scale the screen so that the origin is at its center with the vertical axis running from –2 units of dimensionless distance at the bottom to +2 units at the top. (Note that the side-to-side distance scale will have to be different to preserve the proper "aspect ratio" between horizontal and vertical coordinates. To a reasonable first approximation, you might assume that a "vertical pixel" takes up the same space as a "horizontal pixel." You may need to experiment with this.)

2. With $n = -2$, give the satellite an initial velocity that will put it into circular orbit. (This is a simple check of the process by which the equation of motion was made dimensionless.) Then try initial speeds that are smaller or larger and/or simply directed differently. Does your program give results that make sense and that correspond to the things you know about orbits under Newtonian gravity? Explain why or why not.

3. Now try using different values of *n* for both circular and *especially* noncircular orbits. What happens if $n \le -3$? What happens for large positive values of *n*? What values of *n* give closed orbits? For what values of *n* does an "escape velocity" seem to exist? Can you explain any of your observations analytically?

4. Play with your program and see what else you can discover.