Physics 132

HW #8 Solutions

Professor Mallinckrodt

Assignment: From Problem Set #9: 1, 5, 6, 9, 11.2, 12, 16, 21 & Problem Set #10: 17, 22

9-1 a) $n = N/V = p/kT = 2.45 \times 10^{19}$ molecules/cm^3

b) If 2.45×10^{19} molecules arranged in a cubic structure then each edge has $(2.45 \times 10^{19})^{1/3} = 2.9 \times 10^6$ molecules and the spacing is $(1\text{ cm})/2.9 \times 10^6 = 3.4$ nm.

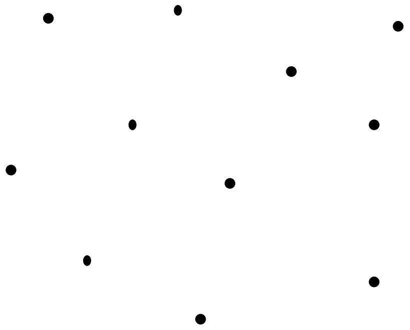
c)

$$N/V = \frac{\text{mass of 1 cm}^3\text{ of carbon/mass of 1 carbon atom}}{1\text{ cm}^3} = \frac{\rho_C}{\text{mass of 1 carbon atom}}$$

$$= \frac{2.27\text{ g/cm}^3}{(12\text{ g}/N_A)} = 1.14 \times 10^{23}\text{ atoms/cm}^3$$

The same calculation as in part b gives the average spacing as 0.21 nm

d) The spacing is about 17 times the diameter. A scaled representation is shown at right with the diameter of the molecules being 1 mm.



e) Using the diameter of the molecules we can see that the scale is 1 mm to 0.21 nm or 4.8 million to 1.

f) $v_{ave} = \sqrt{8 k_B T/\pi m}$ and $m = M/N_A$ with $M = 28$ g (Nitrogen is diatomic) so

$$v_{ave} = \sqrt{8RT/\pi M} = 476\text{ m/s}$$

g) The scale model molecules would be moving 4.8 million times as fast or 2.3×10^9 m/s ~ 8 times the speed of light!

h) With molecules this close together moving *that* fast, collisions should be *very* frequent! Just looking one might guess that a molecule could go on average 10 to 100 average spacings without colliding with another molecule, but that only takes ($170\text{ mm}/2.3 \times 10^9$ m/s) = 74 ps to 740 ps so we might expect 10^9 or 10^{10} collisions per second.

9-5 (S&B 21-17)

a) The book intends for us to consider the pressure constant and just use

$$Q = nC_p\Delta T = \frac{C_p\, pV\, \Delta T}{RT} = 118\text{ kJ}$$

In fact, when we heat the air in a house, we don't change either the pressure *or* the volume of the air in the house. How can the pressure *and* volume remain constant as the temperature increases? The ideal gas law says that this can happen only if the number of molecules goes down and a little thought will show that this is exactly what happens. If our house was *well* sealed, the pressure *would* go up, but what happens instead is that molecules are forced outside through any small gaps. These molecules do work on the outside air as the gas from the house expands against atmospheric pressure, but once they are lost they gain no more energy from the heating process.

To work the problem correctly, you should really answer parts c, d, and e first. So here goes

c and d) In fact, at *any* tmperature $E = nC_vT = \frac{7}{2}\, pV = 35.5$ MJ. Thus, the energy content of the house does not change!

e) The energy left the house *carried* by molecules which also left the house!

a) (revisited) Since the energy content of the gas that remains in the house does not change, you see that the only thing the heat really ends up doing is supplying the energy required for the gas to do work as it expands to the outdoors! By the first law the infiitesimal energy change of the gas that stays *in* the house is

$$dE = dQ - p\, dV$$

where dV represents the volume of the gas that leaks outside. Now, since dE = 0

$$dQ = p\,dV = nR\,dT \quad \text{(by the ideal gas law)}$$

and since the number inside the house at any time is given by pV/RT with p and V constant, we have

$$dQ = pV\,(dT/T)$$

which can be integrated to find the total heat added

$$Q = pV \ln(T\_f/T\_i) = 33.7 \text{ kJ}$$

b) $mgh = Q \qquad m = Q/gh = 6.03 \times 10^3$ kg (if you follow the book's method) or

$= 1.72 \times 10^3$ kg (if you do it correctly!)

9-6 (S&B 21-19)

This is a calorimetry problem. Lets assume that we mix 0.25 L of air at T_{air} = 20°C with 0.75 L of water at T_w = 90°C. Thus we have

$$\Delta E = nC_v(T_{eq} - T_{air}) + mc_w(T_{eq} - T_w) = 0$$

but we have n = pV/RT = 0.0104 moles, $C_v = \frac{5}{2}$ R, m = 0.75 kg, and c_w = 4186 J/mol K, so, solving for $T_{eq,}$ we have

$$T_{eq} = \frac{nC_vT_{air} + mc_wT_w}{nC_v + mc_w} = 89.995\ °C$$

which implies that the temperature dropped by a *very* small amount. We would be on firmer ground if we determined how much heat is required to warm the air essentially to 90°C ...

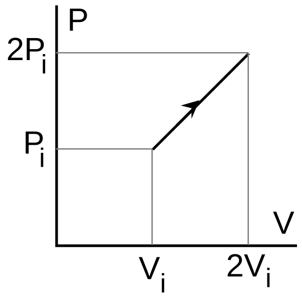
$$Q = nC_v\Delta T = 15.1 \text{ J}$$

... and then calculated how much the temperature of the tea would drop in warming the air by this amount.

$$\Delta T = -Q/mc_w = -0.005\ °C$$

with allowance for slightly varying assumptions, the tea should drop in temperature by between 10^{-3} and 10^{-2} °C.

9-9 (S&B 21-23) b) A sketch of this process is shown at right.



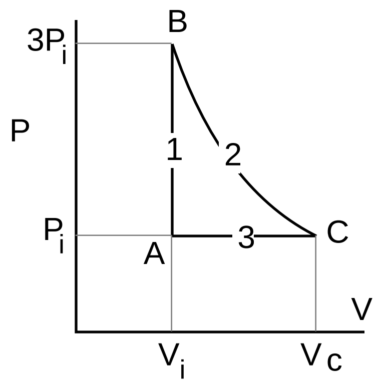
a) The rms speed will double only if the temperature quadruples. Since T PV and P V (P/V = const = P_i/V_i) we find that both P and V must double. Thus we find

$$dQ = dE + dW = nC_v\,dT + P\,dV$$

$$Q = nC_v\Delta T + \int \frac{P_iV\,dV}{V_i} = nC_v(3T_i) + \frac{P_i}{V_i}\frac{1}{2}((2V_i)^2 - V_i{}^2)$$

$$= \frac{15}{2}\ nRT_i + \frac{3}{2}\ nRT_i = 9nRT_i = 9P_iV_i$$

9-11.2 (S&B 21-30) a) A sketch is shown at right



b) $3P_i V_i^{\gamma} = P_i V_C^{\gamma}$ $\quad V_C = 3^{1/\gamma} V_i = 2.19 V_i$

c) $T_B = P_B V_B/nR = 3 P_i V_i/nR = 3T_i$

d) (T at the end of the adiabatic leg)

$T_C = P_C V_C/nR = 3^{1/\gamma} P_i V_i/nR = 3^{1/\gamma} T_i = 2.19 T_i$

e) $W = W_1 + W_2 + W_3$ (see problem 9-12 for W_2)

$$= 0 + \frac{P_B V_B}{\gamma - 1}\left(1 - (V_B/V_C)^{\gamma - 1}\right) - P_C (V_C - V_A)$$

$$= \frac{3P_i V_i}{\gamma - 1}\left(1 - 3^{-(\gamma-1)/\gamma}\right) - P_i V_i \left(3^{1/\gamma} - 1\right)$$

$$= \frac{2 - \gamma(3^{1/\gamma} - 1)}{\gamma - 1} \quad P_i V_i = 0.829\, P_i V_i$$

9-12 (S&B 21-31)

a) Find the work done in the adiabatic expansion

$$W = \int P\, dV = P_i V_i^{\gamma} \int dV/V^{\gamma} = \frac{P_i V_i}{\gamma - 1}\left(1 - (V_i/V_f)^{\gamma - 1}\right)$$

and, with $P_i = 21$ atm (gauge pressure = 20 atm), $V_i = 50\ \mathrm{cm}^3$ and $V_f = 400\ \mathrm{cm}^3$,

$W = 150$ J

This work is done in the time it takes for 1/2 of one revolution since a four stroke “cycle” takes two full revolutions. Thus, $t = \frac{1}{2}\frac{1\ \mathrm{min}}{2500} = 12.0$ ms and

$P_{ave} = W/t = 12.5$ kW

b) $P_{ave} = 16.8$ hp. A diesel engine with four such cylinders would generate 67 hp at this speed which seems quite in line with typical engine powers.

9-16 (S&B 21-37) We find that

$$\text{ratio at 10 km} = \frac{n_N(10\ \mathrm{km})}{n_O(10\ \mathrm{km})} = \frac{n_N(0)\exp(-m_N gh/k_B T)}{n_O(0)\exp(-m_O gh/k_B T)}$$

$$= (\text{ratio at sea level}) \exp((M_O - M_N)gh/RT$$

where $M = N_A m$ and $R = N_A k_B$. Thus

ratio at 10 km = 4.00 exp[(.032 – .028)kg (9.8 N/kg) (10 km)/R(300 K)]

$= 4\, e^{0.157} = 4.68$

9-21 (S&B 21-49) a) Using the formula derived in the text,

$$l = \frac{1}{\sqrt{2}\pi d^2 n_v} = \frac{k_B T}{\sqrt{2}\pi d^2 p} = 93 \text{ nm}$$

b) Solving for p we get

$$p = \frac{k_B T}{\sqrt{2}\pi d^2 l} = 9.5 \times 10^{-3} \text{ Pa} = 9.3 \times 10^{-8} \text{ atm}$$

c) Using the same formula with l = d we get

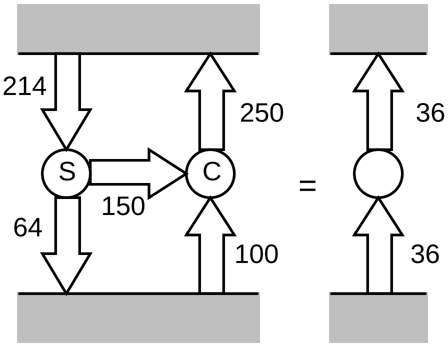
$$p = \frac{k_B T}{\sqrt{2}\pi d^3} = 30.5 \times 10^{6} \text{ Pa} = 302 \text{ atm}$$

d) Since, at this pressure, the molecules are essentially right next to each other, we might expect the gas to be forced to condense to a liquid or solid.

10-17 (S&B 22-17) a) $Q_{in} = W/e = 214$ J, $Q_{out} = Q_{in} - W = 64$ J

b) With the Carnot engine runing in reverse as a heat pump using the 150 J output of S we have a total heat of –36 J *from* the firebox and –36 J *to* the environment. That is, we have built a perfect refrigerator transfering 36 J each cycle from the environment to the firebox in violation of the Clausius statement of the Second law of Thermo.

The two devices and the equivalent device are shown at right.

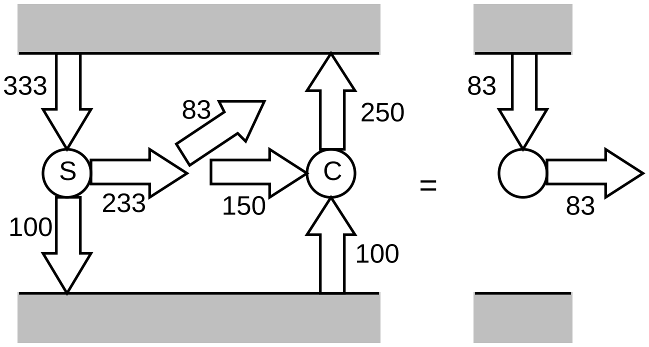


c) $Q_{in} - W = Q_{out} = 100$ J and $W = (0.70)Q_{in}$

$Q_{in} = 333$ J and $W = 233$ J

d) With S operating with 100 J of exhaust each cycle, it supplies 233 J of work each cycle, 150 J of which can be used to run one cycle of the Carnot refrigerator. In this case we have a total heat of 83 J from the firebox, 83 J of work output, and no net heat to the environment. That is, we have built a perfect heat engine transforming 83 J of heat from the firebox to work each cycle with *no* exhaust to the environment in violation of the Kelvin-Planck statement of the Second law of Thermo.

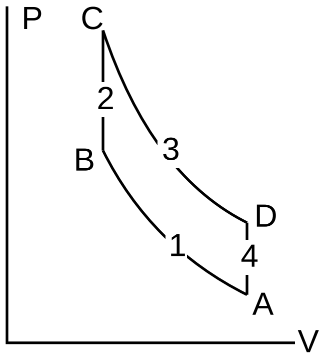
The two devices and the equivalent device are shown at right.



e) In this case the change in entropy of the universe is the change in entropy of the firebox, so

$\Delta S = Q/T = -83 \text{ J}/750 \text{ K} = -0.11$ J/K

10-22 (S&B 22-22) The cycle is shown at right. We are given V_A = 500 cm^3, P_A = 100 kPa, T_A = 293 K, V_A/V_B = 8, T_C = 1023 K, and that the gas is ideal and diatomic.
Thus,



$$V_B = 500\ \text{cm}^3/8 = 62.5\ \text{cm}^3$$

$$P_B = P_A\,(V_A/V_B)^{\gamma} = 1838\ \text{kPa}$$

$$T_B = \frac{P_BV_B}{P_AV_A}\ T_A = 673\ \text{K}$$

$$P_C = P_B(T_C/T_B) = 2793\ \text{kPa}$$

$$P_D = P_C\,(V_C/V_D)^{\gamma} = 152\ \text{kPa}$$

$$T_D = \frac{P_BV_B}{P_AV_A}\ T_A = 673\ \text{K}$$

The energies can be determined from $E = nC_vT = 2.5\ nRT = 2.5\ PV$. We get

State	T (K)	P (kPa)	V(cm^3)	E_{int} (J)
A	293	100	500	125
B	673	1838	62.5	287
C	1023	2793	62.5	436
D	445	152	500	190
A	293	100	500	125

Now we can find the heats and works as follows: For the adiabatic paths, Q = 0 and $W = \frac{P_iV_i}{\gamma - 1}\ [1 - (V_i/V_f)^{\gamma-1}]$. For the isovolumetric paths, W = 0 and $Q = nC_v\Delta T$. We get

Path	Q (J)	W (J)	ΔE_{int} (J)
A → B	0	–162	162
B → C	149	0	149
C → D	0	246	–246
D → A	–65	0	–65
ABCDA	84	84	0