1. a) Sketch graphs of the following three functions representing the position of a particle moving in one dimension as a function of time:

   $x(t) = -3t + 2t^2$ $\quad$ $x(t) = 3 \cos(2t^2 + \pi)$ $\quad$ $x(t) = 2 [1 - \sin(4t - \pi/2)]$

   when all quantities are expressed in SI units.

   b) Which of these represent oscillatory motion? Which represent periodic motion? Which represent simple harmonic motion (SHM)?

   c) For those that represent SHM, find the amplitude, the angular frequency, the frequency, the period, and the first time (t>0) when the particle is *at* the equilibrium position.

2. Characterize the motions of the following objects in terms of whether they are merely oscillatory or strictly periodic and comment on the frequency and amplitude of each one.

   a) A bouncing ball with *and without* energy loss.

   b) A mass on a spring

   c) A garage door

   d) A pendulum

   e) The level of an outdoor mercury thermometer

3.0 Show by direct substitution that *all three* of the following functions,

$x(t) = A \sin(\omega t + \varphi)$ $\quad$ $x(t) = A \cos(\omega t + \varphi)$ $\quad$ $x(t) = A \sin\omega t + B \cos\omega t$

are nontrivial solutions of the differential equation

$$m \frac{d^2}{dt^2} x(t) = -k\, x(t)$$

with arbitrary values for the constants A, B, and φ i*f and only if* $\omega = \pm\sqrt{k/m}$.

3.1 A 1.0 kg mass is attached to a spring with a force constant of 100 N/m and is released at rest 20 cm from its equilibrium position at t = 0.

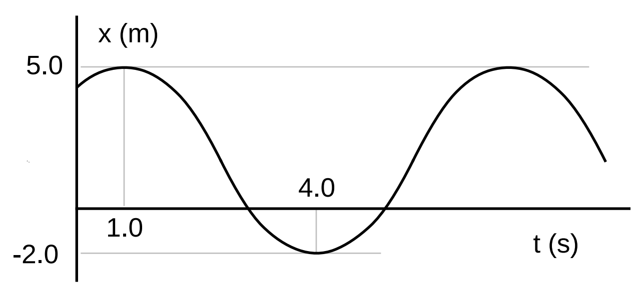
a) Sketch a graph of x(t) with both axes labeled and scaled.

b) Find the values of the constants (A, B, and/or φ) for each of the three functions given in problem 3.0 that will accurately reproduce the sketch you drew in part a.

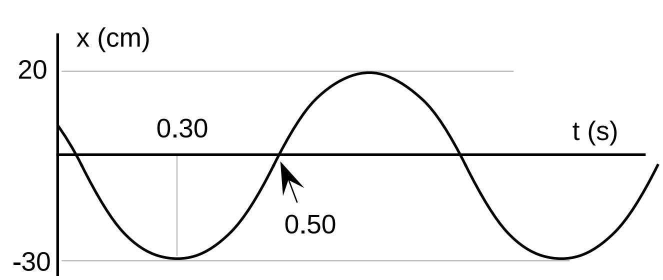
c) Repeat parts a and b for the situation where the mass is at its equilibrium position at t = 0 and given an initial velocity of +50 cm/s.

4.0 a) A particle undergoes SHM about a point *other* than x = 0 as shown in the graph at right. Find its period, amplitude, equilibrium position, and angular frequency.

b) Write the equation for the motion in the form $x(t) = x_{eq} + A \cos(\omega t + \varphi)$. [Hints: You have already found values for x_{eq}, A, and ω. You need only determine the value of φ. At what *time* does x(t) first take on its maximum value? Therefore, what is the value of "the phase" *at* that time?]



4.1 Write the equation for the motion shown in the graph at right. [Hint: This time you will need to get two equations, one for each specific time and solve them simultaneously for ω and φ.]



5. Two particles undergoing SHM have the same amplitude, but the red particle has twice the frequency of the green one.

a) How do the maximum speeds compare? (That is, what is $v_{max\text{-}red}/v_{max\text{-}green}$?)

b) How do the maximum accelerations compare?

c) If the amplitude is 20 cm and the maximum acceleration of the green particle is 1 g (*i.e.*, 9.8 m/s^2), what is the maximum speed of the red particle?

6.0 A particle undergoing SHM reaches its maximum positive acceleration of 5.0 m/s^2 at a time that is 3.0 s before it reaches its maximum speed.

a) What is its angular frequency?

b) What is its amplitude?

c) What is its maximum speed?

6.1 A particle undergoing SHM reaches its maximum positive acceleration of a_{max} at a time τ before it reaches its maximum speed. In terms of a_{max} and τ,

a) what is its angular frequency?

b) what is its amplitude?

c) what is its maximum speed?

7. A 43 g mass is attached to a spring with a force constant of 650 N/m. If $x_i = -12$ cm and $v_i = 27$ cm/s,

a) Find the amplitude and the cosine phase constant. [By "cosine phase constant" I mean the value of φ in $x(t) = A\cos(\omega t + \varphi)$ as opposed, for instance, to its value in $x(t) = A\sin(\omega t + \varphi)$.]

8. A particle of mass y is attached to a spring with a force constant w. It is then placed at its equilibrium position and given a *quick* shove so that it begins oscillating with amplitude β. In terms of the "givens", what was its initial speed *just* after the shove. [Hint: The words "quick shove" should be taken to mean that the particle did not move appreciably *during* the shove. Thus, the particle can be assumed to have started its oscillatory motion *at* the equilibrium position. Apply conservation of energy between the starting point and a subsequent maximum displacement position. Note: Part of the point of this problem and its weird symbols for mass, force constant, and amplitude, is to emphasize that the symbols used in the *general* equations represent *general* values and that, in any *specific* problem, we have *specific* values for those quantities.]

9. a) Show that the total energy, $E(t) = \frac{1}{2}m[v(t)]^2 + \frac{1}{2}k[x(t)]^2$ for a simple harmonic oscillator with $x(t) = A\cos(\omega t + \varphi)$ and $\omega = \sqrt{k/m}$ is *not* a function of time, but is, in fact, *constant* and equal to $\frac{1}{2}kA^2$ and also to $\frac{1}{2}mv_{max}^2$.

b) What is the kinetic energy when the potential energy equals $\frac{1}{2}kA^2$?

c) What is the potential energy when the kinetic energy equals $\frac{1}{2}mv_{max}^2$?

10. a) What is the central feature that makes 1) a mass on a spring, 2) a *small* amplitude simple pendulum, 3) a *small* amplitude physical pendulum, and 4) a torsional pendulum, *all* simple harmonic oscillators?

b) Remind me what it means to *say* that these systems are "simple harmonic oscillators."

c) Why is it necessary to specify that the amplitude is "small" in the case of the case of the simple pendulum and the physical pendulum?

11. a) Show that the period of a physical pendulum is given by $T = 2\pi\sqrt{\frac{I_{cm} + md^2}{mgd}}$ where I_{cm} is the rotational inertia about an axis through the center of mass and perpendicular to the plane of oscillation and d is the distance from the center of mass to the suspension point.

b) Sketch a graph of the period as a function of d.

c) Show that the period has a minimum for $d = \sqrt{I_{cm}/m}$.

d) In terms of d, how long would a *simple* pendulum have to be to have this same period?

12. A particle moves in the x-y plane with simple harmonic motion in *both* dimensions. That is,

$x(t) = A_x \cos(\omega_x t + \varphi_x)$ and $y(t) = A_y \cos(\omega_y t + \varphi_y)$

Describe the path of the particle (with words *and* a drawing) if

a) $A_x = A_y$ and $\omega_x = \omega_y$ and $\varphi_x = 0$ and $\varphi_y = -\pi/2$

b) $A_x = A_y$ and $\omega_x = \omega_y$ and $\varphi_x = 0$ and $\varphi_y = \pi/2$

c) $A_x = A_y$ and $\omega_x = \omega_y$ and $\varphi_x = 0$ and $\varphi_y = 0$

d) $A_x = 2A_y$ and $\omega_x = \omega_y$ and $\varphi_x = 0$ and $\varphi_y = -\pi/2$

e) $A_x = A_y$ and $\omega_x = 2\omega_y$ and $\varphi_x = 0$ and $\varphi_y = -\pi/2$

13. The differential equation (DE) for the linearly damped harmonic oscillator is

$$\frac{d^2}{dt^2}x(t) + \frac{b}{m}\frac{d}{dt}x(t) + \frac{k}{m}x(t) = 0$$

a) Verify by differentiation and direct substitution that this DE is "satisfied by " the function

$$x(t) = A\, e^{-(bt/2m)} \cos(\omega t + \varphi) \qquad \text{with } \omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

b) Explain why this solution becomes unphysical when the damping coefficient gets too large and find the critical value for b at which this becomes an issue.

c) For values of b *larger* than that critical value, verify that the solution to the DE is

$$x(t) = A\, e^{-\alpha t} + B\, e^{-\beta t} \quad \text{with } \alpha, \beta = \frac{b}{2m} \pm \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}}$$ (*i.e.*, use the + sign for α and – for β.)

d) Explain the *physical* difference between the behaviors of the *overdamped* solution of part c and the *underdamped* solution of part a.

14. Suppose it takes 20 full oscillations for the amplitude of a linearly damped harmonic oscillator to be reduced to one half of its initial value. How much is the frequency reduced as a result of the damping? That is, by what percentage is the frequency less than the "natural frequency"?

Hint: The time t is the time when $e^{-bt/2m} = 1/2$ and it is *also* equal to $20\,T = 20\,(2\pi/\omega)$. Use the resulting relationship to find b in terms of k and m.

15. a) Look around you and/or think about the *real* world and identify at least five *real* examples of things that approximate harmonic oscillators under appropriate circumstances.

b) Characterize them in terms of whether or not they are *lightly* or *heavily* damped, *forced* or *unforced*.

c) Give estimates for typical amplitudes and frequencies of each one (or at least *compare* those amplitudes and frequencies to other known amplitudes and frequencies).

1. a) What gravitational force does the Sun exert on the Earth?
   b) What gravitational force does the Earth exert on the Sun?
   c) What is the acceleration of the Earth toward the Sun due to the Sun's force on it?
   d) What is the acceleration of the Sun toward the Earth due to the Earth's force on it?
   e) Explain why it is that, although the Earth and Sun exert precisely *equal* forces on each other, it is primarily the *Earth* that "does the orbiting".

2. a) Which exerts the greater gravitational force on the moon: The Earth or the Sun?
   b) Given your answer to part a, would you expect the Moon to be orbiting the Earth or the Sun?
   c) Explain how one might make sense of all of this. (Hint: What would it *mean* to say that the Moon is *not* orbiting the Sun?)

3. Find the *direction* of the total gravitational force on the Moon due to the Earth and the Sun when
   a) the Moon is on the *same* side of the Earth as the Sun.
   b) the Moon is on the *opposite* side of the Earth from the Sun.
   c) the Moon and Sun form a 90 degree angle with the Earth at the apex.
   d) Clearly indicate the directions found in parts a through c on a diagram of the Moon's orbit.

4. To get an idea of the strength of the gravitational force, determine approximately how long it would take for two people, initially at rest and separated by about 1 m, to move about 1 cm closer together if the *only* force acting on each of them were the gravitational force of attraction from the other. You will have to make several simplifying assumptions; be sure to acknowledge them explicitly in your work.

5. When Henry Cavendish performed the first measurement of the universal gravitational constant G in 1798, he considered himself to be determining the first accurate value for the mass of the Earth. By his time the radius of the Earth was well known so it was already possible to get an estimate for the mass of the Earth from the known density of common rocks which tends to run about 2.8 g/cm^3.
   a) What do you get for the Earth's mass from this info and what assumption do you have to make?
   With Newton's theory of gravitation and a value for the universal gravitational constant G, one can determine the mass of the earth from the value of the gravitational field strength g at the surface.
   b) What do you get for the Earth's mass from this info?
   c) What is the implication of this result given your answer to part a?

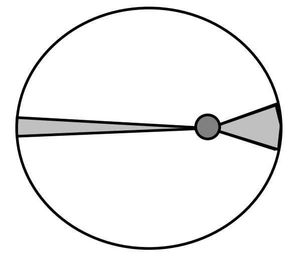
6. a) Calculate the magnitude of the gravitational field strength $\vec{g}$, at the surface of Jupiter and compare it to our familiar value, 9.8 N/kg? (You will need to find the mass and radius of Jupiter.)
   b) Jupiter has a mass that is more than 300 times that of Earth. Why isn't the gravitational field strength a *lot* stronger at its surface than at the surface of Earth?
   c) Suppose we made a big planet with 10 times the radius of Earth, but with the *same* average density. How would the magnitudes of $\vec{g}$ at the surfaces compare? That is, what is $g_{big\ planet}/g_{earth}$?
   d) Jupiter has a radius *more* than 10 times that of Earth, so how do you reconcile your results from parts a and c?

7. Jupiter's orbit about the Sun is nearly circular and has a radius of 5.2 AU. (1 AU—a so-called "Astronomical Unit"—is essentially the radius of the Earth's orbit about the Sun.)
   a) Using only this information and Kepler's Third Law, determine how many years it takes for Jupiter to orbit the Sun.
   b) Mercury takes only 88 days to orbit the Sun, also in a roughly circular orbit. What does Kepler's Third Law predict for its orbital radius?
   c) Halley's comet travels along a *highly* elliptical orbit and returns to the Sun every 76 years. What is its approximate "aphelion" distance—*i.e.*, its maximum distance from the Sun.
   d) Compare your answer to part c with the average orbital radius of Pluto which is about 39.5 AU.

8. The eccentricity of an elliptical orbit can be defined as $\varepsilon \equiv \frac{r_{max} - r_{min}}{r_{max} + r_{min}}$ where r_{max} and r_{min} are the maximum and minimum distances from the central gravitating body—the so-called "periapses." The angular momentum of the orbiting body at any given position is given by $|\mathbf{r} \times \mathbf{p}| = rmv_{perp}$ where v_{perp} is the "tangential component" of its velocity—that is, the component perpendicular to **r**. Suppose that the eccentricity of a certain orbit is 0.50

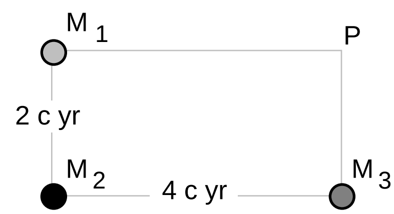
a) Find the ratio of the periapses—that is, r_{max}/r_{min}.

b) Using the fact that the angular momentum is conserved, find the ratio of the *speeds* at the periapses—that is, $v(r_{max})/v(r_{min})$. [Hint: At the periapses, and *only* at the periapses, the velocity *is* perpendicular to **r**.]



c) The figure at right represents the "areas swept out" by the orbiting object in equal times near each periapse. As long as the time interval Δt is small, these areas are essentially triangular and given by $A = \frac{1}{2}$ base x height $= \frac{1}{2}(v\Delta t)\ r$. Using this result and the results of parts a and b, find the ratio of the areas of these triangles—that is, $A(r_{max})/A(r_{min})$—and comment on the relationship of this result to Kepler's Second Law.

9. Three stars are located as shown at three corners of a 2 x 4 light year ("c yr") rectangle in an otherwise empty region of space. Their masses are $M_1 = M_\odot$, $M_2 = 2\,M_\odot$, $M_3 = 0.5\,M_\odot$ where "$M_\odot$" is the symbol representing "one solar mass" and is equal to the mass of our Sun—*i.e.*, 1.99×10^{30} kg. One light year is the distance traveled by light in one year and so is *literally* given by
$1\text{ c yr} = 1\,(2.998 \times 10^{8}\text{ m/s})\,(365.24 \times 24 \times 3600\text{ s}) = 9.461 \times 10^{15}\text{ m}.$



a) What is the gravitational force on the star with mass M_2? Don't forget that force is a *vector*!

b) What is the gravitational field $\vec{g}$ at the empty corner of the rectangle, P? Don't forget that $\vec{g}$ is a *vector*!

c) What is the total gravitational potential energy of the three star system?

d) How much work would have to be done *on* a star with 1 solar mass if it were brought in from *very* far away to the point P? Be sure that you understand what it means that this work is negative! (For the purposes of this calculation, we assume that the fourth star does not interact appreciably with any other matter on its way to the point P and "*very* far away" means far enough that its initial gravitational potential energy is negligible.)

10. Suppose that we drop a rock from some altitude h to the surface of a planet that is *exactly* like Earth except that it has no atmosphere (and, therefore, causes no air resistance to be applied to the falling rock.)

a) If h = 10 m, how fast is the rock moving when it hits the ground?

b) Explain why you can use either the equations of *constant acceleration* kinematics *or* conservation of energy with our old expression $U_g = mgh$ to solve the problem in part a.

c) If h = 2000 km, how fast is the rock moving when it hits the ground? (Use conservation of energy with the appropriate expression for the gravitational potential energy

d) Explain why you *cannot* use either the equations of *constant acceleration* kinematics *or* the expression $U_g = mgh$ to solve the problem in part c.

e) If h is "very large" (by which I mean much, much, (much!) larger than the radius of the planet), how fast is the rock moving when it hits the ground?

f) If one wanted to launch a particle upwards from the ground with a high enough speed so that it would never fall back to the ground, what minimum speed would be required?

11. Suppose a satellite is orbiting the Earth in an elliptical orbit with an eccentricity of 0.80 and a perigee distance (the smallest orbital radius) of 1.00 R_E where R_E is the radius of the Earth. We'd like to find out how fast it is moving at perigee and apogee and compare those speeds with those for *circular* orbits at the same distances. Here we go ...

a) Draw a sketch showing the Earth and the satellite orbit. (This *should* go without saying!)

b) First, use the known eccentricity to find the apogee distance (the *largest* orbital radius). [See a previous problem for the definition of eccentricity.]

c) Next, write the equation that expresses the fact that the total energy of the satellite (kinetic + gravitational potential) is the same at apogee and perigee. [Use the notation r_a and r_p, v_a and v_p, to represent the radius and speed at apogee and perigee respectively.]

d) Now, conservation of angular momentum requires that the product of the orbital radius and the orbital speed at apogee and perigee be the same. Write an equation, using the same notation, that expresses *this* fact.

e) Solve your two equations for v_p and v_a!

f) Find the speeds of satellites in circular orbits with radii r_p and r_a. Call them v_{cp} and v_{ca}.

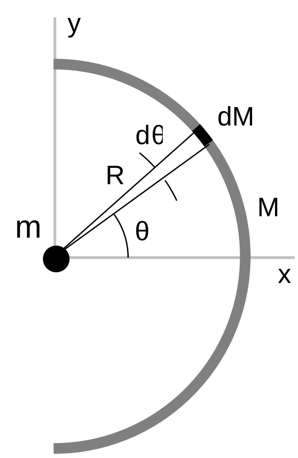
g) By what percentage is the speed of *our* satellite at perigee *greater* than v_{cp}?

h) By what percentage is the speed of *our* satellite at apogee *less* than v_{ca}?

i) Use Kepler's Third Law to find the period of the elliptical orbit. [Hint: You will first want to find the period of a satellite in one of the circular orbits to use in the proportionality.]

Note: If one wants to "boost" the orbital radius of a satellite in circular orbit from r_p to r_a one could—and we always do!—increase its speed (by firing its rockets) by the amount calculated in part g, wait until it reaches apogee, and then increase its speed again to the new circular orbit value. This is called a "Hohmann transfer orbit".

12. A particle of mass m is placed at the center of curvature of a thin semicircular object of radius R with uniformly distributed mass M as shown in the diagram at right. We want to find the net gravitational force on the particle due to the semicircular object. This will involve integration and the steps of this problem are intended primarily to lead you through the process of "setting up" the particular integral to be performed and to help train you in how such a process is done *in general*.



a) Find an expression for the magnitude of the infinitesimal force dF acting on the particle due to a *typical* infinitesimal piece of the semicircular object as shown at right. [Note: It is important that the piece be "typical" and not "special" in any way—such as the piece centered at $\theta = 0$—because we want to develop an expression for its effect that can be used for *any* piece. Note also that this force is indeed "infinitesimal" because it is proportional to the infinitesimal quantity dM.]

b) Make a compelling argument based on the symmetry of the configuration that when all of these infinitesimal forces are added up, the *net* gravitational force on the particle will be *directly* toward the right. That is, that all the vertical force contributions will precisely cancel. [As a result, all we need to do is add up the horizontal *components* of the infinitesimal force contributions.]

c) Find the horizontal component of the infinitesimal force found in part a. [Your expression at this point should be in terms of G, dM, m, R, and θ.]

d) At this point we see that it will be convenient to integrate over θ as it runs from $-\pi/2$ to $+\pi/2$ Accordingly, we want to express dM in terms of the associated infinitesimal angular range dθ (see the figure). We can do this by using the fact that the mass is distributed "uniformly" so that the mass per unit angle is constant and, obviously, given by M/π (since the entire mass M is distributed over an angular range π). Using this info fill in the last step and express dM in terms of M and dθ.

e) At this point you have expressed dF *explicitly* in terms of its dependence on θ and you are ready to "add up" the contributions from all infinitesimal pieces of the semicircular object. Do so by performing the integral.

f) Express your result, the net gravitational force on the particle appropriately as a vector.

g) What is the net gravitational force that the *particle* exerts on the semicircular *object*?

1. Atmospheric pressure is due to the weight of the atmosphere. Indeed, the total force exerted on the surface of the Earth by the atmosphere is equal to that weight.

   a) Find the weight of the atmosphere.

   b) Now, find the mass of the atmosphere.

   c) What assumption did you have to make in part b?

   d) To check on that assumption let's do a calculation based on an oversimplified model. Suppose the atmosphere had a constant density up to some height above which there was a perfect vacuum. Look up the density of air at sea level and use that figure along with your answer to part b to find the required height of this oversimplified model of the atmosphere.

   e) If the atmosphere were composed purely of nitrogen (and it almost *is*), how many atoms of nitrogen would it consist of?

2. For four tires to support a car each one must support approximately a quarter of the weight.

   a) Use this fact along with estimates for the gauge pressure in a typical tire and the area of contact with the ground to obtain an estimate for the weight of a typical car. Does your answer seem reasonable? Explain why or why not.

   b) What happens to the area of contact when the pressure in the tire is low? Why?

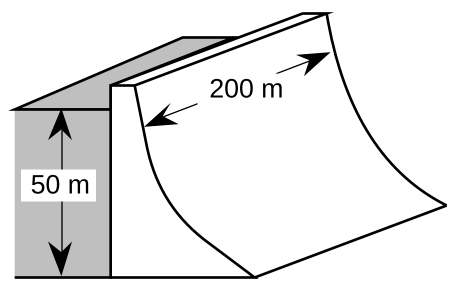
3. One of the first things that a scuba diver learns is that diving to a depth of about 10 m increases the pressure one's body by a factor of two.

   a) Find a more precise value for the depth at which this happens.

   b) Because water is an almost incompressible fluid, the pressure increases by about 1 atmosphere for every additional 10 m of depth. How would this change—and why—if water were more compressible?

4. To get an idea of just how incompressible water is, note that its bulk modulus is 0.21×10^{10} N/m^2. Therefore, by approximately what percentage does the density of water increase at the bottom of an oceanic trench that is 10 km deep? (Hint: Find the percentage volume change of a given initial volume of water when subjected to the pressure found at that depth assuming that water *is* incompressible. Comment on the reasonableness of that assumption when you finish your calculation.)

5. The 200 m wide dam shown at right contains a lake with a depth of 50 m directly behind the dam.

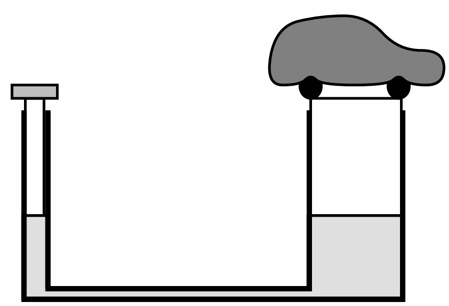


   a) Find the total force that the water exerts on the dam. (You will need to integrate the contributions from an infinite number of infinitesimal horizontal strips each of which has a constant, but different, pressure acting on it.)

   b) Why do you suppose dams are shaped as shown?

   c) Given your answer to part a, does the design of the dam need to take into consideration the volume of water in the lake? That is, would it matter whether the lake was Puddingstone Reservoir or the Pacific Ocean?

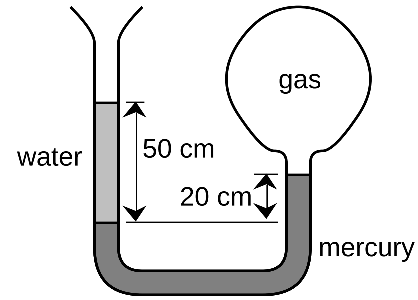
6. Using hydraulic lifts, a small force can be used to lift a *much* larger weight. Suppose you find that the weight of your 1.0 kg textbook is enough to support a 2500 kg UAV ("Urban Assault Vehicle") using the hydraulic lift shown at right.



   a) If the diameter of the "slave cylinder" (the one that supports the car) is 25 cm, what is the diameter of the "master cylinder" (the one on which your textbook is resting)? Assume that the pistons have negligible mass.

   b) How does your answer change if the piston in the master cylinder has a mass of 1.0 kg and the one in the slave cylinder has a mass of 500 kg?

7. The open tube manometer at right is filled with mercury and water as shown and being used as a gauge for the pressure in the gas. What *is* the gauge pressure of the gas?



8. In a compressible fluid like the atmosphere, the density is a function of the pressure. One can still write $\Delta p = \rho g h$ as long as h is very small (so that Δp is very small and, therefore, ρ *doesn't* change very much.) These considerations lead to the differential expression

$$dp = \rho(p)\, g\, dy$$

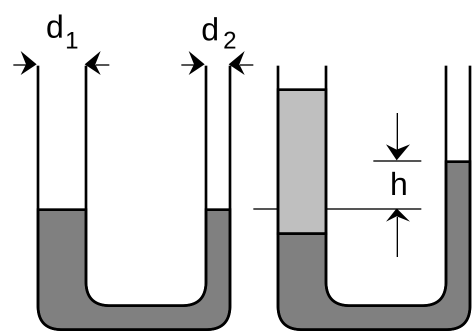
where $\rho(p)$ simply makes explicit the dependence of ρ on p, dy is an infinitesimal change in height, and dp is the corresponding change in the pressure. We can separate variables (by dividing both sides by $\rho(p)$) and then directly integrate this expression (on the right from "ground level" (h=0) to any other height h and on the left from the pressure p_o at "ground level" to the pressure p at the height h getting

$$\int_{p_o}^{p(h)} \frac{dp}{\rho(p)} = \int_0^h g\, dy = gh$$

a) Now suppose that $\rho(p) = \alpha p$. That is, the density is directly proportional to the pressure with a proportionality constant α. (This should seem plausible for very compressible fluids like gases.) Perform the integration and solve for p(h)—the pressure as a function of altitude.

b) Assuming that this density function reasonably models the behavior of the atmosphere, your result should too. *Does* your result seem reasonable? Explain why or why not.

9. A U-tube having arms of diameter d_1 = 2.0 cm and d_2 = 1.0 cm is filled with mercury to the level shown in the first sketch at the right. Then 50 cm^3 of water is poured into the first arm. How high above the original level does the mercury in the second arm rise as a result? [Hint: There are three conditions that must be met 1) the volume of mercury lost on the left must equal the volume of mercury gained on the right, 2) the pressure at the bottom of the water column must be the same as the pressure in the other arm at the same height, 3) the height of the water column is determined by its volume and the cross sectional area.



10. A 30 cm x 20 cm x 5 cm plank of wood with a density of 700 kg/m^3 floats in a pool of water.

a) How high does its top surface float above the surface of the water?

b) If a 1.00 kg block of copper were attached to the top of the plank, show that the plank would not be able to keep it dry.

c) Would the plank-copper combo sink to the bottom?

11. A spherical Helium balloon 12 meters in diameter uses 350 kg of material just for the balloon itself.

a) About how large a payload (basket, passengers, etc.) can the balloon lift at sea level? (Note: The pressure inside the balloon is essentially the same as outside.)

b) As the balloon rises the pressure drops and so does the density of the air and the helium (unless it is not allowed to expand). Briefly discuss the advantages and/or disadvantages of allowing the helium to expand by using a balloon that is not fully filled at ground level.

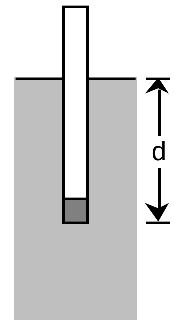
12. To obtain the "weight under water" of an object, it is suspended by a string that hangs from a spring scale and then fully submerged in water. Compare the weights under water of: An 800 g gold crown and a 1.2 kg aluminum lawn table.

13. Water projected vertically out of a 4.0 mm diameter nozzle at the end of a lawn hose reaches a maximum height of 9.0 m.

a) How fast is the water moving as it leaves the nozzle?

b) How fast is the water moving inside the 2.0 cm diameter hose itself?

14. A hollow tube of length L and cross-sectional area A is plugged at one end with a weight so that it floats vertically in water (density ρ_w) with a length d that is submerged as shown at right.



a) Show that, if the tube is pushed down a distance x, the buoyant force increases causing a net upward force given by $\rho_w gxA$ that tends to restore the tube to its equilibrium position.

b) Show that, when the tube rises a distance x, the buoyant force decreases causing a net downward force given by $\rho_w gxA$ that tends to restore the tube to its equilibrium position.

c) Explain why this leads to simple harmonic motion and show that the period is given by $T = 2\pi\sqrt{d/g}$. [Hint: Use Archimedes' principle to find the total mass of the tube + weight system in terms of A, d, and ρ_w.]

d) We have ignored viscosity in this analysis. What would its effect be on the result?

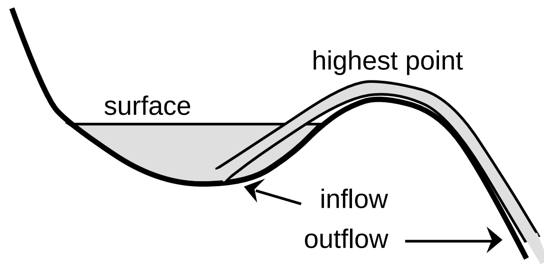
15. Suppose that a hose with a 2.0 cm diameter can fill a 20 liter bucket in 30 seconds.

a) What is the water speed in the hose itself?

b) Ignoring viscosity and other nonideal effects, what is the water speed in the 18 cm diameter water "main" (i.e., "big pipe") that ultimately supplies the hose?

c) Given that the pressure near the end of the hose is essentially atmospheric pressure, what is the *gauge* pressure in the water main?

16. The siphon tube shown at right has a constant cross sectional area and will drain the lake if one can 1) fill the tube with water in the first place, 2) keep the outflow end lower than the inflow end at the bottom of the pond, and 3) make sure that the highest part of the siphon tube is not *too* high.



a) Use Bernoulli's equation to get relationships between fluid speeds, fluid pressures, and heights of the four points—surface, inflow, highest point, outflow.

b) Given that the pressure at the outflow end is atmospheric, find the flow speed in the tube in terms of the vertical displacement $h_{surface} - h_{outflow}$.

c) Use that result to show that the tube outflow end must be kept below the tube inflow end, if you want to drain the pond.

d) Find the pressure at the highest point in terms of atmospheric pressure and the vertical displacement $h_{highest} - h_{outflow}$.

e) Use that result to show that the highest point must be no more than about 10 m above the outflow point. (What happens if it is higher than that?)

Answers to Numerical Questions

1. a) 5.2×10^{19} N, b) 5.3×10^{18} kg, d) 8.0 km, e) 2.3×10^{44}

3. a) 10.3 m

4. 4.7%

5. 3.5×10^{9} N

6. a) 5.0 mm, b) Increases to 6.5 mm

7. -2.18×10^{4} Pa (= –.215 atm = –163 mmHg)

9. 9.36 mm

10. a) 1.5 cm

11. a) 655 kg

12. The crown "weighs" 7.43 N and the lawn chair "weighs" 7.40 N

13. a) 13.3 m/s, b) .531 m/s

15. a) 2.12 m/s, b) 2.62 cm/s, c) 2.25 kPa

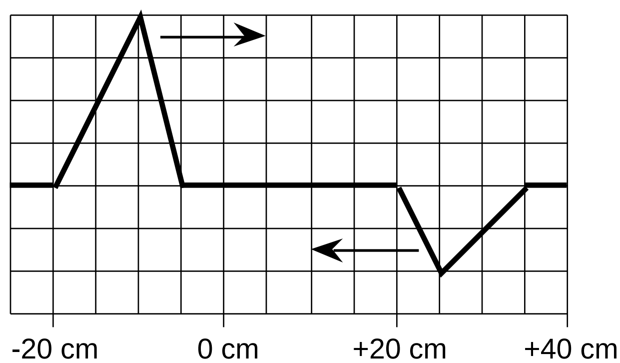
1. A transverse wave pulse is given by $A\,e^{-b(x+ct)^2}$ with A = 5.0 mm, b = 4.0 m^{-2}, and c = a constant. At t = 20 μs, the pulse peak is located at x = –8.0 cm.
   a) What is the wave speed for this pulse?
   b) What is the value of the constant c?
   c) Sketch the shape of the pulse $y(x)$ at t = 0. Be sure to label and scale your axes.
   d) Explain how you know that the *shape* of this pulse does not change with time.

2. A transverse wave pulse $y(x,t)$ has a shape at t = 0 given by $y(x,t=0) = \dfrac{6.0 \times 10^{-2}}{x^2 + 4x + 7}$ when x and y are given in meters.
   a) At what value of x is the peak of the pulse located at t = 0? (Include the units.)
   b) What is the height of the pulse peak? (Include the units.)
   c) What is the value of y at positions 1.0 meter to either side of the pulse peak? (Include units.)
   d) Sketch the shape of this wave pulse.

   Two seconds later the shape of the pulse is given by $y(x,t=2.0\text{ s}) = \dfrac{6.0 \times 10^{-2}}{x^2 - 12x + 39}$ when x and y are given in meters.

   e) Where is the peak located at t = 2.0 s?
   f) Redo part b for the new shape and show that the answer is the same.
   g) Redo part c for the new shape and show that the answer is the same.
   h) How fast is the pulse moving?
   i) Write the general formula—good for *any* time t—for $y(x,t)$ with x and y in meters and t in seconds.

3. The two transverse wave pulses shown at right travel along the same string with equal and opposite speeds of 25 cm/s. Sketch the appearance of the string at eight consecutive 0.10 second intervals beginning 0.40 seconds after the time shown in the figure.



4.0 The transverse disturbance associated with a periodic wave on a string is given by

$$y(x,t) = (5.2\text{ cm})\sin(0.38\text{ m}^{-1}\,x + 232\text{ μs}^{-1}\,t)$$

Find its amplitude, wavelength, period, frequency, wave speed, and direction of propagation (*i.e.*, +x or –x).

4.1 The transverse disturbance associated with a periodic wave on a string is given by

$y(x,t) = A\sin(kx + \omega t)$ (with A, k, and ω being positive constants.)

Find its amplitude, wavelength, period, frequency, wave speed, and direction of propagation (*i.e.*, +x or –x).

4.2 The transverse disturbance associated with a periodic wave on a string has an amplitude of 2.1 cm, a wavelength of 34 cm, and a wave speed of 130 m/s in the +x direction. Determine the formula for $y(x,t)$.

5. The high E string on an electric guitar is a 78 cm long, solid steel wire with a diameter of 0.23 mm. It is adjusted to a tension of 174 N. The density of steel is 7.9 g/cm^3.
   a) What is the linear mass density (*i.e.*, the mass per unit length) of this string?
   b) What is the wave speed on this string?
   c) At what frequency will pulses make *round trips* between the ends of this string?

6. Two different strings or ropes are spliced together (with a *totally* unobtrusive splice!) and held taut.

   a) Why is the wave speed guaranteed to be larger in the lighter weight string?

   b) Sketch what happens when a pulse travels across the boundary from the lighter weight string to the heavier weight string.

   c) Do the same thing for the opposite case.

   d) Redo parts b and c for the case that the heavier string is *way* heavier (and I mean *way*, *way* heavier!!) than the lighter string. Explain the difference between *this* case and parts b and c.

   e) Redo parts b and c for the case that the two strings are identical. Again, explain the difference.

7. A sinusoidal wave on a string makes pieces of the string move transversely with an amplitude of 3.0 mm at a frequency of 400 Hz. When one piece is at its maximum positive transverse displacement, the pieces 20 cm to either side are at their equilibrium positions. The tension in the string is 150 N. We want to know what power is being transmitted by the wave. We'll just calculate *whatever* we can until we know enough to do so!

   a) Find the wavelength of the wave.

   b) Find the wave speed.

   c) Find the linear mass density of the string.

   d) Find the power being transmitted.

8. What happens to the power being delivered by a given sinusoidal wave on a given string if

   a) the wave amplitude is doubled (keeping the frequency the same)?

   b) the wave frequency is cut in half (keeping the amplitude the same)?

   c) the string is made four times as taut (keeping its linear density the same)?

   d) The diameter of the string is doubled (keeping the tension the same)

9. A rope of length L with linear mass density μ is hung from the ceiling. The tension in the rope at any point is simply equal to the weight of the portion hanging below it.

   a) Obtain an expression for the tension in the rope as a function of x, the distance from the bottom.

   b) Use your result from part a to find an expression for the wave speed as a function of x.

   c) Find the infinitesimal time dt that it will take for a pulse to move an infinitesimal distance dx centered at some position x.

   d) "Add up" all of those dt's to find the total time it would take a pulse to move from the bottom end of the rope to the ceiling. Your answer can only depend on μ, L, and g.

   e) If you accept what I just said in part d, explain why your expression can *not* depend on μ? (Hint: Think about the dimensions of the various involved quantities.)

   f) Evaluate your expression from part d for a 2.0 m long rope hung from the ceiling of a room on the third planet out from the Sun.

10. A wave on a string with mass density μ damped by frictional losses *in* the string might be described by the equation $y(x,t) = A_o e^{-bx} \sin(kx - \omega t)$.

   a) Sketch a "snapshot" of this wave. That is, what the wave looks like at some specific time.

   b) Explain the effect of the exponential term in this expression.

   c) What is the rate at which power is transmitted to the right at some specific value of x.

   d) Explain how your answer to part c is consistent with the idea that energy is being dissipated within the string itself.

Answers to Numerical Questions

1. a) 4.0 km/s, b) 4.0 km/s

2. a) x = -2.0 m, b) 3.0 cm, c) 1.5 cm at both places, e) x = 6.0 m, h) 4.0 m/s

4.0 a) amplitude = 5.2 cm, wavelength = 16.5 m, period = 2.7×10^{-8} s, frequency = 3.7×10^{7} Hz, wavespeed = 6.1×10^{8} m/s (oops! v > speed of light!!), direction is toward –x.

5. a) 3.28×10^{-4} kg/m, b) 728 m/s, c) 467 Hz

7. a) 80 cm, b) 320 m/s, c) 1.46 g/m, d) 13.3 W

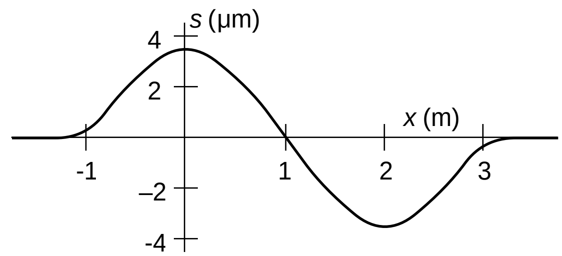
9. f) 0.904 s

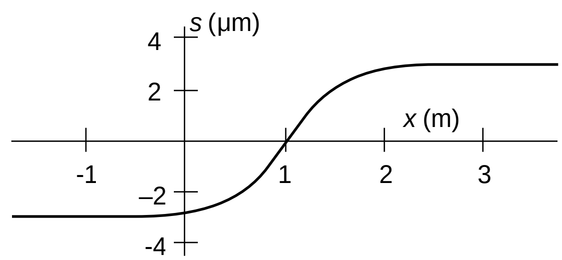
Problem Set #5 (Chapter 17) — Physics 132, Professor Mallinckrodt

1. As we will see later, the bulk modulus of air for high frequency compressional oscillations like those produced by sound is given to a good approximation by $\frac{7}{5}$(atmospheric pressure) with the factor of $\frac{7}{5}$ being a result of the fact that air is primarily composed of diatomic molecules! Use this information to calculate the speed of sound under standard conditions .

2. The density of air is inversely proportional to the so-called "absolute" temperature which can be specified using the Kelvin scale. On the Kelvin scale the lowest possible temperature is assigned the value 0K and the temperature at which water freezes is 273K, a result of the fact that one degree on the Kelvin scale is equal in "size" to one degree on the Celsius scale (whatever *that* means and we'll get back to *that* later!) Using this information, show that the speed of sound at standard atmospheric pressure is given by $v(T_C) = v(T_C = 0)\sqrt{1 + \frac{T_C}{273°\text{C}}}$.

3. The disturbance associated with a longitudinal wave like a sound wave can be described (at one specific *time*) in terms of the longitudinal displacement *s* of molecules as a function of their equilibrium position *x*. A graph of *s*(*x*) requires *careful* interpretation. Explain in *detail* what the first graph of *s*(*x*) at the right *means*. For instance:





a) At what position or positions are molecules displaced furthest to the left?

b) What does it *mean* that the scale on the *s* axis is in µm's while that on the *x* axis is in m's?

c) What does it *mean* that *s* is positive to the left of *x* = 1 m and negative to the right of *x* = 1 m?

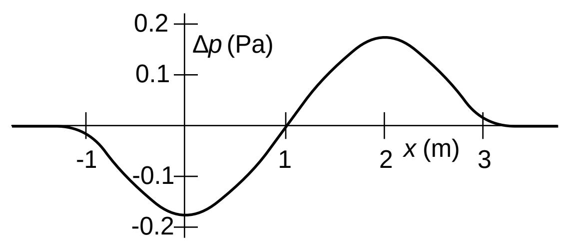
d) What does this imply about the density of molecules—and, therefore, about the change in pressure Δp near *x* = 1 m?

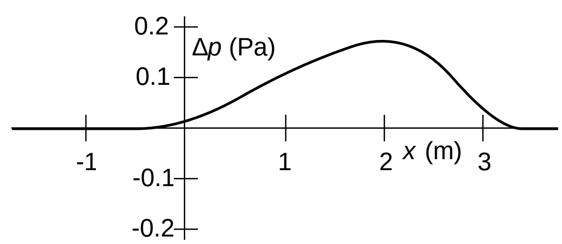
e) Sketch the pressure profile $\Delta p(x)$that would accompany this displacement profile.

f) Can you tell which direction this wave is propagating?

g) Discuss the implications of the second graph at right, addressing similar questions.

4. The disturbance associated with a longitudinal wave like a sound wave can be described (at one specific *time*) in terms of the *change* in pressure from its normal (or "ambient") value as a function of position. Explain in *detail* what the first graph of $\Delta p(x)$ at the right *means*. For instance:





a) What does it *mean* that Δp is negative at x = 0?

b) What are the approximate *absolute* pressures *x* = 0 m, 1 m and 2 m and what do you learn from your result?

c) What longitudinal motions would be required near x = 0 m to produce a negative value for Δp?

d) Sketch the displacement profile *s*(*x*) that would accompany this pressure profile.

e) Can you tell me which direction this wave is propagating?

f) Discuss the implications of the second graph at right, addressing similar questions.

5.1 Use the fact that Δp is proportional to $-\frac{\partial s}{\partial x}$ to sketch the pressure profiles—*i.e.* $\Delta p(x)$—for the two plots of $s(x)$ in problem 3 and comment on whether or not your sketches agree with the more qualitative analysis you did in that problem.

5.2 Use the fact that s is proportional to $-\int \Delta p \, dx$ to sketch the displacement profile—*i.e.* $s(x)$—for the two plots of $\Delta p(x)$ in problem 3 and comment on whether or not your sketches agree with the more qualitative analysis you did in that problem.

6. a.) Look up the bulk modulus and density for both copper and aluminum and use them to calculate the speed of sound in both materials.

b) You should have found that the bulk modulus of copper is about twice that of aluminum. Why then, is the speed of sound greater in aluminum?

7. A sinusoidal sound wave with a frequency of 1.00 kHz has a sound level of 0 dB. (This is at the so-called "threshold of hearing".)

a) Find the associated pressure and displacement amplitudes.

b) How does the displacement amplitude compare with the diameter of a typical atom which is about 10^{-10} m?

c) Redo part a for a sound level of 140 dB. (For reference, 130 dB is often referred to as the "threshold of pain.")

d) By what factor are your answers to part c greater than your answers to part a? (Do you see *why* your answer is $\sqrt{10^{(140/10)}}$?)

e) Compare the pressure amplitude from part c with atmospheric pressure. What does this comparison tell you about sound waves?

f) How would your answers to parts a and c change if the frequency had been 10 times less—*i.e.*, 100 Hz in stead of 1.00 kHz?

8. Show that the following equations are "dimensionally consistent":

a) $v = \sqrt{B/\rho}$

b) $\Delta p_{max} = \rho v \omega s_{max}$

c) $I = \frac{1}{2}\rho v(\omega s_{max})^2$

9. Suppose that 2.0 meters away from a *very* loud speaker, the sound level is 110 dB.

a) Assuming that the speaker emits sound waves "isotropically" (meaning uniformly in all directions), what is the acoustical power output of the speaker?

b) Your answer to part b *should* surprise you if you are thinking! Why?

c) At what rate is energy entering your ear? (Assume that your ear gathers sound energy from a total effective area of about 1 cm^2.)

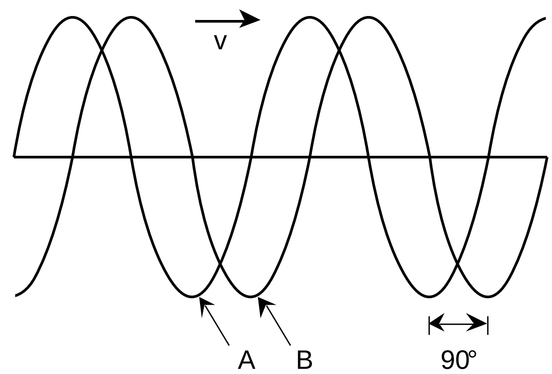
d) What would the sound level be if you moved to a position *twice* as far away from the speaker?

10. Which produces the larger Doppler shift for a given speed: Motion of the source or motion of the observer?

11. As a train passes by the frequency of its horn decreases by 15.0%. How fast is the train going?

Answers to Numerical Questions

1. 332 m/s
6. a) v_{Cu} = 4.0 km/s, v_{Al} = 5.1 km/s
7. a) Δp_{max} = 29 μPa, $s_{max} = 1.1 \times 10^{-11}$ m, c) p_{max} = 290 Pa, $s_{max} = 1.1 \times 10^{-4}$ m, d) Both are larger by a factor of 10^7, f) Δp_{max} would be the same; s_{max} would be greater by a factor of 10.
9. a) 5.0 W, c) 10 μW, d) 104 dB ("Six dB down")
11. 27.8 m/s



1. a) Two equal amplitude, equal frequency sinusoidal waves travel in the *same* direction with a constant phase difference of 90° as shown at right. What does the superposition principle say about the resultant wave function. Describe it in as much quantitative detail as possible *and* sketch the result on a graph like that shown at right.

   b) How does your result change if the phase difference is 45°? 135°? 0°? 180°? 270°? Again, be as *specific* as possible.

   c) What if the two waves in part a had *different* amplitudes, but still the same frequencies? To be concrete, let's suppose that the wave labeled "A" was larger in amplitude by 50% and the wave labeled "B" was smaller by 50%. Without doing any math, sketch the resultant wave and describe the differences between it and your answer to part a.

2. Two identical, in phase sources of sinusoidal sound waves with a frequency of 800 Hz are located 4.0 m ("source A") and 5.0 m ("source B") away from your location along *different* lines.

   a) Draw a picture showing the described set up.

   b) Express the distances to the sources in terms of the *number of wavelengths* that would "fit" in each. (Use v_{sound} = 343 m/s.)

   c) Consider a *specific* time (call it $t = 0$) at which each source emits a *peak* positive pressure. Now, at *that* time consider all the other positions along the lines from each source to your location at which there are also pressure maxima. Draw a graphic representation showing—to scale—the locations of all these pressure maxima.

   d) Describe how your picture would change as time goes on.

   e) At what time will the *next* positive pressure peak reach you from *each* of the two sources?

   f) Sketch a graph showing how the pressure at your location from source A alone would depend on time. Be sure to get the starting "phase" of the oscillation correct so that the first pressure maximum occurs at the proper time as calculated in the previous part.

   g) On the same graph sketch the contribution from source B.

   h) Are the two contributions "in phase"? How do you explain this given the fact that the two *sources are* in phase?

   i) By what fraction of a cycle are the two contributions out of phase?

   j) By how many wavelengths do the two source distances differ?

   k) Subtract off any integer number of wavelengths from your answer to the last part and compare the fractional remainder to your answer from part i.

   l) Why are (or, at least, *should*) your answers to parts i and k be identical?

3. Two identical, in phase sources of sinusoidal sound waves with a frequency of 800 Hz are located on the y-axis at y = 0.00 m and y = 3.00 m. Use v_{sound} = 343 m/s.

   a) Find the *first three locations* along the positive x-axis at which the two sounds add in phase.

   b) How many locations *are* there along the positive x-axis at which the two sounds add in phase?

4. Two identical, in phase sources of sinusoidal sound waves with a frequency of 500 Hz are located on the y-axis at y = –1.00 m and y = +1.00 m. Consider observers located along the line x = 2.00 m.

   a) At how many locations along that line will observers hear relatively loud sound due to constructive interference? (Use v_{sound} = 343 m/s.)

   b) Obtain an equation that could be solved for the y coordinate of the n^{th} such location in the first quadrant counting the one on the x-axis as n = 0.

   c) (Extra credit!!) *Solve* your equation for the n = 1 and n = 2 locations.

5. A 62 cm long steel guitar string is intended to have a fundamental frequency of 82 Hz. Tight strings tend to resonate better, but the guitar neck is built to support a tension of no more than 250 N in each string.
   a) What is the maximum possible diameter of this string?
   b) At other frequencies will the string resonate at when it is "tuned" to 165 Hz?

6. A long tube closed at the lower end is filled with water up to a certain level leaving a column of air that resonates at 500 Hz with 3 pressure antinodes.
   a) Sketch the standing wave pattern for pressure changes in the air-filled part of the tube as described above.
   b) Sketch the standing wave pattern for longitudinal air molecule displacements for the same situation and explain the difference.
   c) At what other frequencies would this column of air resonate?
   d) How long is the column of air? (Use v_{sound} = 343 m/s.)
   e) If you continued pouring water into the tube you'd find other lengths that would resonate at 500 Hz. What are they?

7. Two guitarists are "tuning up" by matching the frequencies of their guitar strings. They both pluck a string and then one of them adjusts the tension. Suppose guitarist A is already tuned to a frequency of 330 Hz and that they hear a beat frequency of 2.0 Hz.
   a) What are the possible frequencies of guitarist B's string?
   b) Guitarist B now tightens her string and the beat frequency rises to 3.0 Hz. What is the frequency of her string now?
   c) By what percentage must guitarist B now change the tension in her string in order to bring the two strings into tune?

8. A person walks toward a wall at 1.6 m/s blowing a horn at a frequency of 330 Hz. Assuming he can hear the reflected sound interfering with the direct sound, what is the beat frequency?

9. A musical octave consists of two pitches that have a frequency ratio of 2:1. The "equal tempered" musical scale breaks each octave up into 12 "chromatic steps" (e.g., C, C#, D, D#, E, ... etc.) with each adjacent pair having a frequency ratio of $2^{1/12}$:1 = 1.0595:1. When playing a guitar, one places the fingers behind frets which progressively shorten the string without changing its tension appreciably.
   a) How much shorter would a string have to be to have a frequency that is one chromatic step higher? (Express your answer as a percentage of the original string length.)
   b) Why do the frets on a guitar get closer together as you go up the neck?

10. The pitch of a pipe organ depends on the temperature. By what percentage do the frequencies of the pipes change when the temperature increases from 15°C to 25°C.

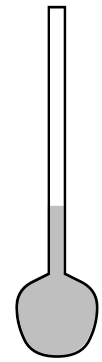
11. The ear is most sensitive to frequencies around 3 kHz as a result of a fundamental resonance of the ear canal. Determine the approximate length of the ear canal from this fact. Does your answer make sense?

Answers to Numerical Questions

2. b) to A 9.33 λ, to B 11.66 λ, e) from A 412 μs, from B 827 μs, i) 0.332 cycle, j) 2.332 λ, k) 0.332 λ
3. a) x= 0.463 m, 1.03 m, 1.77 m, b) 6 (not counting the "one" at x = ∞.)
4. a) 5, c) y = 0.807 m and y = 2.01 m
5. a) 1.97 mm (using density of steel = 7.9 g/cm ^{3}), b) 164 Hz, 246 Hz, 328 Hz ... n(82 Hz), n = 1,2,3, ...
6. c) 100 Hz, 300 Hz, 700 Hz, 900 Hz, ... n(100 Hz), n = odd integer, d) 85.8 cm, e) 51.5 cm, 17.2 cm
7. a) 328 Hz and 332 Hz, b) 333 Hz, c) lower by 1.81%
8. 3.1 Hz
9. a) 5.61% shorter
10. 1.72%
11. 2.9 cm

1. The word "heat" is defined in physics as a transfer of energy from one object to another object as a result of a temperature difference between the two objects. The word "work" was defined earlier and is also a transfer of energy from one object to another. Specifically how then, does "heat" *differ* from "work"?

2.0 Suppose that two thermometers are made using two *different* liquids. In the usual way, the liquids are allowed to expand from a reservoir into a long uniform vertical tube as illustrated at right. The thermometers are then placed in thermal contact with ice water and allowed to come to thermal equilibrium. Marks are made showing the levels of the fluids in the two tubes and those marks are labeled "0°C". Next, the thermometers are placed in thermal contact with boiling water at standard atmospheric pressure and again allowed to come to thermal equilibrium. Again marks are made showing the levels of the fluids in the two tubes and those marks are labeled "100°C". Finally, a series of marks are made along the tubes dividing the distances between the 0°C and the 100°C marks into 100 equal intervals.



a) Make sketches showing the two resulting thermometers. Make it clear in your sketches that the bulbs, the tubes, the amounts of fluid, and the levels of the two primary calibration marks might *all* be different.

b) Suppose the two liquids are now placed in thermal contact with a body at some intermediate temperature. Suppose further that, when everything comes to thermal equilibrium, thermometer A "reads" a "temperature" of "45° C" and thermometer B "reads" a "temperature" of "55° C". Explain why this might happen *even* if the construction and calibration procedures were carried out *perfectly*.

c) Is there any way then, using these two thermometers, to determine what the Celsius temperature of the body actually *is*?

d) If the liquids in the two thermometers had been the *same* could the scenario of part b have happened? In *that* case, would we be able to depend on the readings to give accurate Celsius temperatures?

e) Considering now just a single thermometer of this type, what would happen if the tube were *not* "uniform", that is, if it were, for instance, "tapered" (meaning that the cross-sectional area would decrease or increase as one moves along the length vertically) or if it simply had a section that was particularly narrow or wide? To be specific, explain how the levels of the 100 individual temperature marks might have to be adjusted if the tube gets more and more narrow at higher levels.

f) Could you use this effect to "correct" the thermometers of parts a and b? In particular, what qualitative changes would you make to the "taper" of the tube for thermometer A to *make* it agree with thermometer B? (Assume here that you do *not* make any adjustments to the *levels* of the temperature marks as described in part e.)

g) After all of these considerations, can you explain what a temperature of 50°C actually *means*? Critique the statement, "50°C is half way between the temperatures of the ice and steam points of water."

2.1 Consider the calibration procedure for constant volume gas thermometers and how the pressure vs. temperature calibration graph is used.

a) Suppose the ice and steam pressures happened to be 2.52×10^5 Pa and 3.37×10^5 Pa respectively. What temperature would a pressure of 2.14×10^5 Pa indicate?

b) What does this thermometer give for the Celsius temperature of "absolute zero"? (Note: The result you will get is *not* the result that *actual* constant volume gas thermometers give!)

Experimentally—we find that two *different* constant volume gas thermometers (different gases, different volumes, different pressures) give the *same* temperature readings.

c) Explain why this experimental result *should* be somewhat surprising especially in light of the results of problem 2.0.

d) We also find *experimentally* that all constant volume gas thermometers give the *same* value for the temperature of "absolute zero"—*i.e.*, –273.15°C. Explain why this fact alone is evidence that there is something *fundamentally* "correct" about temperatures determined with such thermometers.

3. a) What single temperature is expressed as the same number of degrees Celsius and Fahrenheit?

   b) Convert the *temperature* 1° C to the corresponding *temperature* on the Fahrenheit scale.

   c) A *change* in temperature of 1° on the Celsius scale would correspond to how large a *change* in temperature on the Fahrenheit scale?

   d) What lesson is intended to be learned from parts b and c?

4. Suppose we devise a new temperature scale (the "Pomona" scale) based on the average temperatures in Pomona on January 1 (defined as 0°P) and July 1 (defined as 100°P). A constant volume gas thermometer shows a pressure of 1.210 atm at 0°P and 1.293 atm at 100°P.

   a) Suppose that on one hot summer day, the same constant volume gas thermometer gives a pressure reading of 1.310 atm. What is the temperature on the Pomona scale?

   b) What is the temperature of absolute zero on the Pomona scale?

   Now, suppose that the ice point of water is determined to be –85°P.

   c) What does this imply for the average temperature in Pomona on July 1 expressed in °C? In °F? (Hint: First determine the "size" of a Pomona degree relative to that of a Celsius degree using the temperatures of the ice point of water and absolute zero.)

   d) What makes the Pomona scale *impractical* (as opposed to simply less *attractive*) to use?

5. A composite rod is constructed from four rods lengths x, 2x, 3x, and 4x attached end to end. The four rods have thermal coefficients of linear expansion 5a, a, 3a, and 2a respectively. What is the thermal coefficient of linear expansion (in terms of a) for the composite rod? (Hint: Find out how much its length changes when the temperature changes by an amount ΔT.)

6. a) Using the standard equation for thermal length change, what is the percentage change in the length of a rod made from a substance with a thermal coefficient of linear expansion of 2.0×10^{-5}/°C when the temperature increases by 10°C?

   b) What do you get for the percentage change in length if the temperature change is 2000°C?

   In fact, when the temperature change is large, we may need to take into account both the changing length of the object itself and the fact that α may be a function of temperature. This is easy to do; we simply write $dL/L = \alpha(T)dT$ and integrate over the required range.

   c) Assuming that α is, in fact, independent of temperature, redo part b by performing the integration. How different is your answer from that obtained in part b?

7. A 60 cm long steel guitar string suspended between two points on a rigid aluminum base vibrates at 200 Hz in its fundamental mode. Now the temperature is raised by 5° Celsius. What is the resulting percentage change in the frequency? (Hints: You will need to consider the *stress* that develops in the string as a result of the thermal *strain*. The length of the string changes by a very small amount so, while you may *not* set $\Delta l = 0$, you *may* say that $l_i \approx l_f$! Similarly you may assume that the density, the linear density, and the cross sectional area of the string are essentially constant.)

8. It is not uncommon for the Space Shuttle to find itself in an environment where the pressure is about 2×10^{-12} atm and the density is about 10^4 particles/cm^3.

   a) What is the temperature in such a region of space? (Be careful with your units!)

   b) Why do you suppose the Space Shuttle doesn't melt?

9. What is the linear coefficient of volume expansion for an ideal gas at a temperature of 300 K? (Hint: Use the ideal gas law to find dV/V.)

10. For an ideal gas, what happens to the

    a) pressure if I increase the temperature by 20% while holding the volume constant?

    b) temperature if I decrease the volume by 50% while doubling the pressure?

    c) density if I double the pressure and increase the temperature by 100%?

Answers to (most of the) Numerical Questions

2.1 a) –44.7°C, b) –296.5°C	5. 2.4a	8. a) 1.47×10^6K
3. Sorry, it'd spoil the fun!	6. a) 0.020%, b) 4.00%, c) 4.08%	9. 3.33×10^{-3}/K
4. a) 120°P, b) –1458°P, c) 36.8°C, 98.2°F	7. 1.4%	10. Sorry!!

1. At Niagara Falls water drops approximately 50 m first turning gravitational potential energy into bulk translational kinetic energy and then "dissipating" that energy internally when it reaches the bottom. a) How many joules of gravitational potential energy does each gram of water lose?

   b) How many calories is this equivalent to?

   c) By how much should the temperature of the water rise as a result of going over the falls?

2. A very moderate day hike might involve a vertical rise of several hundred meters. In climbing, you do work against gravity, calling upon energy reserves that you have accumulated by eating.

   a) About how much energy do you have to expend to move your body upward through such a distance? (Here we are ignoring the fact that a lot of energy is "wasted" metabolically so this is a bare minimum. Depending upon your metabolic efficiency, the actual energy expended might be three to five times the amount you calculate here.)

   b) How many calories of energy reserves would your body use on such a hike?

3. The base metabolic rate of an average human body (*i.e.*, the rate at which stored energy is "burned" when at rest) is approximately 80 W. How many calories do you "burn" each day just sitting there? How many Calories is that?

4. a) What is the final temperature when you mix 5.0 l of water at 20°C with 20.0 l of water at 50°C?

   b) Where is the center of mass of a lopsided barbell with a 5.0 kg weight at x = 20 cm and a 20.0 kg weight at x = 50 cm?

   c) Why did I ask that last question?

5. A 100.0 g aluminum calorimeter contains and is in thermal equilibrium with 30.0 g of water at 20.0°C. Now 200.0 g of a metallic substance with a temperature of 100.0°C is placed in the calorimeter. When the system comes to equilibrium, the final temperature is 41.1°C. What is the most likely identity of the metallic substance?

6. Some houses now use "on demand" water heaters to heat water *at* the point of use *when* it is required. This saves energy previously wasted due to thermal losses from water heater tanks and long pipe runs. Suppose one wants to design such a system to take 15°C water at the input and to produce a 6 liter/min flow of 50°C water using an electric heater. What is the minimum power requirement for the electric heater? (Hint: How much heat must it deliver in one minute?)

7. To cool a soft drink you add a lot of ice expecting some of it to melt as the drink cools to approximately 0° C. Assuming that the drink starts at 20°C and that the ice is essentially at its melting temperature when it is added to the drink, by approximately what percentage will your drink be diluted with water?

8. The polar ice caps are evidence that the oceans of the world are not in thermal equilibrium. Suppose that the oceans *did* come to thermal equilibrium, melting the polar ice caps in the process. Then the oceans would, on average, be cooler.

   a) Estimate the amount of heat required to melt the polar ice caps. (Assume that both have a radius of 1000 km, are 200 m thick, and that they have a temperature just under 0°C.)

   b) Assuming that the oceans and the ice caps form an "isolated system" (meaning that they only exchange energy with each other), estimate the associated temperature drop of the oceans. (Assume that the oceans cover 3/4 of the Earth's surface and have an average depth of 3 km.

   c) What are the implications of your result for part b? Consider particularly the potential effect of an overall global temperature increase of 1°C.

9. Suppose that two lead bullets, each having an initial temperature of 20°C and traveling at 500 m/s, collide head on forming a single lump of hot lead. Assuming that no energy is lost from the two bullet system, describe the final thermodynamic state of that lump of lead?

10. 200 g of gold at 140°C is cooled by adding 100 g of ice at –20°C. Assuming the two substances exchange energy only with each other, describe the final thermodynamic state of the system.

11.1 Calculate the work done by 1 mole of an ideal gas at an initial temperature of 300 K when

a) it doubles its volume isothermally.

b) it doubles its volume isobarically.

c) it doubles its volume while keeping P/V constant.

d) it doubles its volume while keeping PV^{γ} constant. (Express *this* answer in terms of γ.)

e) Evaluate your answer to part d for the case $\gamma = 5/3$.

f) Compare your answer to part c with that for part d when $\gamma = -1$.

g) Why didn't I ask for the work done when the gas doubles its volume isovolumetrically?

11.2 Calculate the final temperature in parts a through e of problem 11.1.

12. On graphs of P vs. V, P vs. T, and V vs. T sketch and label curves representing isobaric, isothermal, and isovolumetric processes.

13. A quantity of ideal gas is trapped in a long vertical cylindrical tube underneath a freely moving piston which compresses the gas under its own weight. The diameter of the cylinder is 12.0 cm; the mass of the piston is 3.0 kg; the temperature of the gas is 20°C; and the bottom of the piston is 40 cm above the base of the tube.

a) How much gas is trapped in the tube?

b) How much work will be done if the temperature is quasistatically raised to 200°C?

c) Suppose that the diameter of the cylinder had been half as great, *i.e.* 6.0 cm. Repeat the calculations of parts a and b and comment on the results.

14. Suppose that 0.35 moles of a gas—not necessarily ideal—at an initial temperature of 300 K expands isobarically from 5.0 liters to 12.0 liters while its internal energy increases by 2200 J.

a) How much heat was transferred to the gas during the process?

Now suppose that the gas is isovolumetrically cooled back to 300 K and that 800 J of heat is transferred to the surroundings in the process. Following this process 800 J of work are done to compress the gas isothermally back to a volume of 5.0 liters.

b) On a P-V diagram, draw a sketch of the cyclical, quasistatic thermodynamic "path" that the gas has followed.

c) During the isothermal compression, how much heat was transferred *to* the gas?

15. A thermodynamic system undergoes a process during which it performs 700 J of work *on* its surroundings while absorbing 1100 J in the form of heat *from* its surroundings.

a) By how much did the energy of the system change during this process?

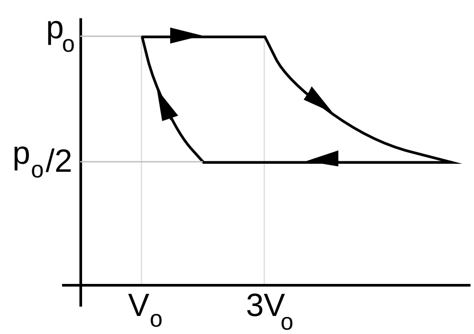
The system is now returned to its original thermodynamic state via a process during which it evolves 700 J in the form of heat *to* its surroundings. (Usage note: The word "evolves" as it is used here *means* "gives off" or "emits.")

b) By how much did the energy of the system change during this (second) process?

c) Find the work done *by* the system *on* its surroundings during this process.

d) What is the significance of the algebraic sign of your answer to part b?

16. A sample of an ideal gas is taken through the thermodynamic cycle shown on the P-V diagram at right. Two of the "legs" of the cycle are isobaric and the other two are isothermal.



a) Find the net work done by the gas along the high pressure isobaric leg in terms of p_o and V_o.

b) Find the net work done *by* the gas in one complete cycle in terms of p_o and V_o.

c) Find the net heat supplied *to* the gas in one cycle in terms of p_o and V_o.

17. A (not necessarily ideal) gas is adiabatically compressed to a pressure of 5.0 atmospheres. Then it is isobarically expanded to a volume that is 3.0 liters larger. Finally, it is isovolumetrically cooled to its initial thermodynamic state. Along one of these legs 2000 J of heat is transferred *to* the surroundings. The *net* work done *by* the gas along all three legs is 100 J.

    a) Sketch this three "leg" cycle on a P-V diagram.

    b) Determine the values of Q, W, and ΔE along each one of the three legs.

18. A 2.0 kg block of aluminum is heated at atmospheric pressure from 20°C to 120°C.

    a) Find the values of Q, W, and ΔE during this process. (Hint: Recall that, for a solid, the *volume* coefficient of expansion is *three* times the linear coefficient of expansion.)

    b) Based on your calculations for part a, explain why we can often equate Q and ΔE for thermodynamic processes involving solids?

19. A copper rod 2.0 cm in diameter and 1.0 m long conducts heat from a region where the temperature is 400°C to your hand (at 38°C) at the other end.

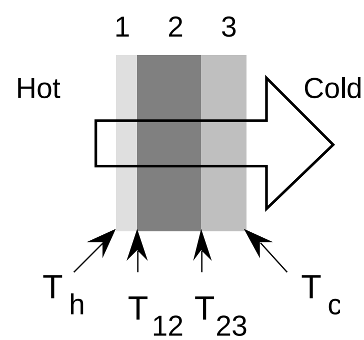
    a) At what rate does heat reach your hand?

    b) Why might you prefer the rod to be made of iron?

20. a) Which is the greater source of heat loss for a house: A small single pane window with a total area of 0.30 m^2 made of glass 6.0 mm thick (pretty thick glass!) or a large 20 m x 3 m wall made of wood 20 cm thick?

    b) In reality, a window is surrounded by air that is significantly cooled on the hot side and significantly warmed on the cold side so the temperature difference across it may be far less than across the wall. What effect would this have on your answer to part a?

21. A wall is made of multiple layers of materials each with its own thickness and thermal conductivity as shown at right. At equilibrium, the rate at which heat flows through each layer must be the same. (Otherwise, the energy near the interfaces would be increasing or decreasing and the temperatures would be changing in violation of the assumption of equilibrium!)



    a) Write expressions for the rates of heat flow (H_1, H_2, H_3) through each layer in terms of the thermal conductivities (k_1, k_2, k_3), the temperature differences across each layer (ΔT_1, ΔT_2, ΔT_3), the thicknesses (L_1, L_2, L_3), and the *common* area A.

    b) Recognizing that $H_1 = H_2 = H_3 \equiv H$ and solving for the individual temperature differences, get an expression for the *total* temperature difference ($T_h - T_c$) in terms of H, A, and the thermal conductivities and thicknesses.

    c) Show that $H = \dfrac{A(T_h - T_c)}{\sum_i L_i/k_i}$ where, in this case, the subscript i runs from 1 to 3.

    d) Suppose that the three layers are (from the hot side to the cold side) 20 cm of aluminum, 5.0 mm of asbestos, and 2.0 cm of concrete and that T_h = 30°C and T_c = –10°C. Find the rate of heat flow per unit area, H/A, *and* the two intermediate temperatures T_{12} and T_{23}.

    e) The asbestos layer is by far the thinnest so why does it have such a large ΔT?

    f) The aluminum layer is by far the thickest so why is T_{12} so close to T_h?

    g) How would removing the aluminum layer affect the overall heat flow through this "wall"?

22. The thermal conductivity of glass is 0.80 W/m°C.

    a) Find the R value of 3.2 mm thick window glass in m^2°C/W.

    b) Convert your answer in part a to the more standard units for R values—*i.e.*, ft^2 °F h/Btu.

23. Suppose an electric hot water heater contains 50 gallons of water and is in the shape of a cylinder 3 ft in diameter and 3 ft high. The temperature is controlled by a thermostat that turns the heater on when the temperature drops to 150°F and turns it off when the temperature reaches 160°F. When no hot water is being used, 3 hours elapse between heating cycles.

    a) How much does each heating cycle cost us at $.12/kWh? (Hints: One gallon of water weighs 8.3 lbs on Earth. Calculate the required energy in Btu's and convert to kWh. Assume 100% heating efficiency.)

b) What is the approximate R value of the container walls? (Explain any reasonable assumptions you might have to make.)

c) Suppose we fully wrap the water heater with a 3" fiberglass "blanket" having an R value of 10 ft^2 °F h/Btu. How long should it now go between heating cycles?

d) How much money should we save each month as a result of installing the blanket?

(Note: The effectiveness of a fiberglass blanket is easily compromised when it does not fully surround the heater. Heat can escape through the uncovered bottom; if there are any gaps, convection currents can increase the rate of heat loss; etc.)

24. When the space shuttle is in orbit and on the night side of the Earth, half of it exchanges radiation with the Earth at around 300 K and the other half with outer space at an effective temperature of about 3 K. Assuming that the total surface area of the shuttle is 1000 m^2 and that its surface has an emissivity of 0.40 and a temperature of 280 K ...

a) What net power does it receive from the Earth, what net power does it radiate to outer space, and what is the net rate at which it loses or gains energy due to thermal radiation?

b) How do your answers change if the shuttle's surface temperature is 250K?

c) What Temperature would make the net radiated power be zero?

25. The temperature of the Earth is determined largely by the requirement that the energy it receives from the Sun is equal to the energy it radiates to outer space. The Sun's disk occupies a fraction f = 5 x 10^{-6} of the sky and is at a temperature of about 6000K. The rest of the sky is at an effective temperature of about 3K. Thus we get power received = power emitted or

$$f\,\sigma Ae(Tsun^4 - Tearth^4) = \sigma Ae(Tearth^4 - Tspace^4) \qquad f\,(Tsun^4 - Tearth^4) = Tearth^4 - Tspace^4$$

a) Solve the equation for T_{earth} in °C. (Note, the greenhouse effect adds another 10 or 15°C on average to your result.)

b) Mars is 1.52 times as far away from the Sun as Earth. Accordingly the Sun occupies a fraction of the sky that is $(1.52)^2$ times *smaller* than it does from Earth. There is no greenhouse effect on Mars and its average temperature is about -40°C. What does the calculation give?

Answers to (most of the) Numerical Questions

1. a) 0.49 J, b.) 0.12 cal, c) 0.12°C
2. a) 2 or 3 x 10^5 J, b) 2 or 3 x 10^5 cal (folding in the efficiency)
3. 1.6 x 10^6 cal = 1600 Cal
4. a) 44°C, b) 44 cm
5. copper
6. 14.6 kW
7. 25%
8. a) 4 x 10^{23} J, b) 0.09°C
9. Molten lead at 1400°C ($c_{liq\ Pb}$ = 172J/kg°C)
10. Ice and gold at -2.4°C

11.1 a) 1.73 kJ, b) 2.49 kJ, c) 3.74 kJ,
d) $\left(\frac{2^{1-\gamma} - 1}{1-\gamma}\right)$ 2.49 kJ, e) 1.38 kJ

11.2 a) 300 K, b) 600 K, c) 1200 K,
d) $(2^{1-\gamma})$ 300 K, e) 189 K

13. a) 4.8 x 10^{-3} moles, b) 7.2 J, c) Same
14. a) 3.42 kJ, c) -2.20 kJ
15. a) +400 J, b) –400 J, c) –300 J
17. b) Leg 1: Q = 0, W = -1420 J, ΔE = 1420 J; Leg 2: Q = 2100 J, W = 1520 J, ΔE = 580 J; Leg 3: ΔE = Q = –2000 J, W = 0
18. b) Q = 180 kJ, W = 5.4 mJ, ΔE = 180 kJ
19. a) 45 W
21. d) H/A = 450 W/m^2, T_{12} = 29.6°C, T_{23} = 1.2°C
22. a) 4.0 x 10^{-3} m^2°C/W, b) 0.023 ft^2°F h/Btu
23. a) $0.15, b) 2.5-3 ft^2°F h/Btu, c) 13-15 h, d) $25 to $30.
24. a) 22.2 kW from Earth, 69.7 kW to space, 47.5 kW net loss, b) 47.6 kW from Earth, 44.3 kW to space, 3.3 kW net gain, c) 252 K
25. a) 11°C, b) –43°C

1. To get a feel for the microscopic properties of a gas, let's look at the properties of the most familiar one—air—and construct a scale model. Air is composed almost entirely of nitrogen so we will simplify our model by considering a pure nitrogen gas at a temperature of 300 K and standard atmospheric pressure. We will treat it as an ideal gas—not too bad an approximation under these circumstances.

   a) What is the number density of this gas in atoms/cm^3?

   b) What is the average distance between the atoms? (Hint: Consider the atoms to be arranged in a regular grid and calculate the required spacing in all three spatial directions to obtain the number density calculated in part a.)

   c) Redo the calculations of parts a and b for solid carbon (whose atoms have nearly the same size and mass as nitrogen) in the graphite form. The density of graphite is 2.27 g/cm^3.

   d) Assuming that the spacing obtained from the results for graphite gives you a reasonable approximation for the size of a carbon or nitrogen or oxygen atom, about how many times the size of a nitrogen atom is the distance between the atoms in air? Draw a scale model showing several atoms with the atoms having a diameter of 1 mm.

   e) What is the scale of this model? (By scale I mean the size of a scaled object relative to its actual size. Express in the standard form "x to 1".)

   f) What is the average speed of the atoms in the nitrogen gas? Compare to the speed of sound in air.

   g) Given the scale factor for your model, how fast would the scale model atoms be moving? Compare to the speed of light, 3.0×10^8 m/s.

   h) Given the average spacing and the speed of the atoms, would you expect collisions to be rare or frequent? Can you estimate a collision frequency?

2. In fact air is composed of many different gases so let's consider the effect of *mixing* gases. Suppose we mix 0.020 moles of helium with 0.010 moles of argon in a 1.0 liter container and let it come to thermal equilibrium at 300 K.

   a) How fast are the atoms of each species moving on average?

   b) What is the average energy of the atoms of each species?

   c) What theorem does your answer to part b illustrate?

   Consider a wall of the container that is being struck by atoms of both species. The fact that helium atoms can collide not only with themselves but with argon atoms means that argon atoms can prevent helium atoms headed toward a wall from striking the wall. On the other hand, they are just as likely to collide with a helium atom that has *just* struck the wall, turn it around, and make it strike the wall again. We certainly can't keep track of the nearly 20 billion trillion atoms in the container, but—by the same token—we *can* make *very* precise statistical arguments. In this case, it turns out that, statistically, such collisions force *just* as many atoms to strike the wall *twice* in any given time period as they *prevent* from striking the wall at all. The net result is that the helium atoms collide with the wall *just* as many times with *just* as much force as they would have if the argon molecules were not present at all. Of course, the same result holds for the argon and any other gas that might be present.

   d) Calculate the "partial pressure" attributable to collisions by both species.

   e) What is the ratio of these "partial pressures"? How does it compare to the ratio of the corresponding number densities?

   f) What is the *total* pressure in the gas?

   g) What would you have obtained for the pressure in the gas if you had only known that there were 0.030 moles of gas without knowing the particular species? What lessons do you learn from all of this?

3. Air is about 1000 (perhaps *several* thousand) times less dense than common solids and liquids. What does this fact alone tell you about the average spacing between air molecules compared to their size?

4. Serway and Beichner, Chapter 21, Problem 14
   Consider the piston to be freely movable so that it maintains a constant pressure in the gas.

5. Serway and Beichner, Chapter 21, Problem 17 and *add* ...
   c) Assuming that the pressure remains constant at standard atmospheric pressure, calculate the total energy content of the air in the house when the temperature is 300 K.
   d) Do the same thing for a temperature of 310 K.
   e) What happened to all the energy you used to heat the air?

6. Serway and Beichner, Chapter 21, Problem 19

7. Serway and Beichner, Chapter 21, Problem 20 and *add* ...
   b) Repeat the calculation if the two steps are done in reversed order, that is, constant volume first, constant pressure second.
   c) Show the two processes on a PV diagram and use it to explain why the answers are different.

8. Serway and Beichner, Chapter 21, Problem 21 and *add* ...
   b) Sketch the process on a PV diagram
   c) Explain why the final temperature is different from the initial temperature.

9. Serway and Beichner, Chapter 21, Problem 23 and *add* ...
   b) Sketch the process on a PV diagram.

10. Serway and Beichner, Chapter 21, Problem 26

11.1 Serway and Beichner, Chapter 21, Problem 29 but *change* part d to
   d) Find the temperature at the end of the adiabatic expansion

11.2 Serway and Beichner, Chapter 21, Problem 30 but *change* part d to
   d) Find the temperature at the end of the adiabatic expansion

12. Serway and Beichner, Chapter 21, Problem 31 and *add* ...
   b) Express your answer in horsepower and comment on its reasonableness.

13. Serway and Beichner, Chapter 21, Problem 32

14. Serway and Beichner, Chapter 21, Problem 35 and *add* ...
   At what temperature would this correspond to the average rotational kinetic energy? (Don't forget that diatomic molecules have two rotational degrees of freedom.)

15. Serway and Beichner, Chapter 21, Problem 36 and *add* ...
    Explain why storing energy in the first excited state is essentially not an option at room temperature, but becomes a possibility as the temperature increases. Also explain how this is related to the fact that vibrational motions in diatomic molecules are effectively not counted as "degrees of freedom" at room temperature.

16. Serway and Beichner, Chapter 21, Problem 37
    Note: Assume also that at sea level the ratio is 4 N_2 molecules for every O_2 molecule.

17. Serway and Beichner, Chapter 21, Problem 39

18. Serway and Beichner, Chapter 21, Problem 41

19. Serway and Beichner, Chapter 21, Problem 44
    Note: Assume that the latent heat represents the minimum kinetic energy that molecules must have to "escape" the liquid, that is, the kinetic energy that will leave them with nothing after they have emerged from the liquid into the region above the liquid gas interface.

20. Serway and Beichner, Chapter 21, Problem 46

21. Serway and Beichner, Chapter 21, Problem 49 and *add* ...
    d) What would you expect to happen somewhere around the pressure determined in part c?

Answers to (most of the) Numerical Questions

1. a) 2.4 x 10^{19} atoms/cm^3, b) 3.4 nm,
   c) 1.1 x 10^{23} atoms/cm^3, 0.21 nm
   d) 17 times, e) 4.8 million to 1,
   f) 731 m/s, g) 3.5 x 10^9 m/s (> c!!)
2. a) Helium: 1.37 x 10^3 m/s, Argon: 433 m/s,
   b) Both: 6.21 x 10^{-21} J, d) Helium: 49.8 kPa,
   Argon: 24.9 kPa, e) 2 to 1, same, f) 74.8 kPa,
   g) same
4. 7.52 liters
5. a) 118 kJ, b) 6.03 x 10^3 kg, c & d) 25.3 M
6. Between 10^{-3} and 10^{-2}°C
7. a) $\frac{27}{2}$ PV, b) $\frac{31}{2}$ PV
8. a) 316 K, b) 200 J
9. a) $9P_iV_i$
10. a) 51.5 cm^3, b) 287°C, c) 2.4°C

11.1 b) 8.77 liters, c) 900 K, d) 658 K, e) 336 J

11.2 (Work 11.1 and then generalize!)

12. a) 12.5 kW (the book is wrong), b) 16.8 hp. (To find the *overall average* power, these numbers must be divided by four.)
14. 2.32 x 10^{-21} J
15. @ 0°C, none; at 10,000°C 2.7 x 10^{20}
16. $n_v(N_2)/n_v(O_2) = 4.69$
17. a) 6.80 m/s, b) 7.41 m/s, c) 7.00 m/s
19. a) 7.26 x 10^{-20} J, b) 2.20 km/s, 3.5 x 10^3 K
20. a) 600 light years, 1 billion years,
    b) 5.7 billion km, 1 thousand years.
21. a) 93 nm, b) 9.3 x 10^{-8} atm, c) 302 atm

Note: *All* problems this time are from Chapter 22 of the text (Serway and Beichner, 5th Edition).
To minimize confusion, I am listing the problems by their number *in the text.*

4. ... and add,
   c) If it performs 500 cycles per minute, what is the rate at which it expels exhaust heat?

6. ... and add
   b) If one were *not* trying to produce useful work in the process, how many grams of the solid mercury could be melted with the heat extracted in freezing one gram of the molten aluminum?

8. Be sure you understand that “20% of the maximum possible efficiency” does *not* mean “20% efficiency.” Note: In this problem, like many others, S&B use the word “energy” where they should be saying “energy in the form of heat” or “thermal energy input.” The question here is about how much heat transfer there is *to* the system *from* the 200° reservoir.

11. ... and add
   c) If this engine is using 1 mole of a diatomic ideal gas, what is the ratio of the largest volume it occupies to the smallest volume it occupies in a single cycle. (Tips: Draw a PV diagram and use what you know about isothermal and adiabatic processes along with the ideal gas law to find the ratios of final t initial volumes for each of the expansion legs.)

12. Pretty straightforward!!

13. ... and add
   b) If the power plant produces 1000 MW of power in the summer (pretty typical for a nuclear power plant), how much more energy will it produce each day in the winter assuming that it uses heat *input* at the same rate summer and winter? Express your answer in kWh.

17. Note that the Carnot engine is being run as a refrigerator, transferring heat from the cold reservoir to the hot reservoir using energy in the form of work (150 J each cycle) that is supplied to it by engine S.

18. This is just another cyclic process problem. Make abundant use of the ideal gas law, the first law of thermodynamics, the definition of work, the molar specific heat capacities, and what you know about isothermal and adiabatic processes.

20. Hint: The “fraction of the fuel wasted” can be obtained by considering the amount of work output per cycle under ideal conditions and comparing it with what is actually output assuming the same heat energy input (*i.e.*, the same amount of fuel used) in both cases.

22. Like problem 18, this is just another cyclic process problem. Make abundant use of the ideal gas law, the first law of thermodynamics, the definition of work, the molar specific heat capacities, and, in this case what you know about isovolumetric and adiabatic processes.

23. + b) What happens to the COP when either the high temperature gets higher or the low temperature gets lower/

26. Note that the problem hypothesizes a heat pump that falls *far* short of the theoretical best performance but that it still puts more heat into the room than the energy you “pay for”!

27. Note that a Carnot heat engine operating between these two temperatures would be *highly* efficient and it is precisely for *that* reason that the coefficient of performance for the refrigerator based on the reversed cycle is so low!

30. + c) If the refrigerator is removing 8000 kJ/h from the cold compartment, why isn't the compartment getting colder!

31. + b) Is it a violation of the second law of thermodynamics that the entropy of the water decreases? If so, then what’s up with that? If not, why not?

33. Because the temperature is changing, you need to do an integral here. $\Delta S = \int dS$

36. Here we are looking at an *irreversible* process. We must *construct* a reversible process that yields the same result in order to calculate the entropy change. Thus, we imagine slowly removing heat from the horse shoe as its temperature drops to the final equilibrium temperature *and* slowly adding heat to the water as its temperature rises. The result is *two* calculations like that done in problem 33. (To determine the equilibrium temperature, just solve the ordinary calorimetry problem.)

39. This is an irreversible process so you must construct a reversible process to reach the same final state. Treat the hydrogen as an ideal gas. In that case, how does the final temperature compare to the initial temperature? Thus, what kind of reversible process seems most obvious to use?

43. Notice that they don't specify the process. Use whatever process you like! This problem emphasizes that entropy is a *state* function. Will you get the same result if you use another process?

44. + Do this using two different processes: Constant volume followed by constant pressure *and* constant pressure followed by constant volume. Do you see why it makes no difference?

53. Make liberal use of the ideal gas law to express quantities in terms of n, R, and T_i.

54. + b) What is the maximum theoretical COP for this situation?

60. Make liberal use of the ideal gas law.

63. Make liberal use of the ideal gas law.

Answers to (most of the) Numerical Questions

4. a) 667 J b) 467 J, c) 233 kW

6. a) 50.4%, b) 33.6 g

8. 197 kJ

11. 741 J, b) 459 J, c) 1.60

12. 546 °C

13. a) 33.0%, b) 7.3×10^5 kWh

17. a) 214 J in, 64 J out,
b) Firebox out –36 J, environment in –36 J,
c) 333 J in, 233 J work out,
d) Firebox out 83 J, work out 83 J,
environment in 0 J, e) –0.11 J/K.

18. If you don't *use* the Carnot results *in* your calculations, you can use them to *check* your calculations!

20. a) 51.2%, b) 36.2%

22. Here's a couple of 'em to help you check your work: P_D = 152 kPa, W_{CD} = 246 J

23. a) 9.0

26. 1.17 J

27. 72.2 J

30. 733 W

31. –610 J/K

33. 195 J/K

36. 717 J/K

39. 5.76 J/K

43. 18.4 J/K

44. 34.6 J/K

53. a) $2nRT_i \ln 2$, b) 0.273

54. 78 W

60. a) 4.10 kJ, b) 14.2 kJ, c) 10.1 kJ,
d) 0.288

63. a) 10.5 nRT_i, b) 8.50 nRT_i,
c) 0.190, d) 0.833